Babeş Bolyai University, Cluj Napoca

Faculty of Mathematics and Computer Science

Abstract

Detecting Equilibria in Game Theory – Evolutionary Approach

PhD student:
Tudor-Dan Mihoc

Supervisor:
Prof. D. Dumitrescu

Keywords: Game theory, Nash equilibria, Joint Nash Pareto equilibria, evolutionary methods

Published Work Associated with Thesis

Indexed ISI Thomson

D. Dumitrescu, R. I. Lung and **T. D. Mihoc**: *Meta-Rationality in Normal Form Games*, International Journal of Computers, Communications & Control V(5), volume V, 693–700, December 2010.

D. Dumitrescu, R. I. Lung and **T. D. Mihoc**: *Evolutionary equilibria detection in non-cooperative games*, Applications of Evolutionary Computing, M. Giacobini, et. all, editors, *Lecture Notes in Computer Science*, volume 5484, 253–262. Springer Berlin/Heidelberg, (EVOSTAR 2009), 2009.

R. I. Lung, **T.D. Mihoc** and D. Dumitrescu: *Nash equilibria detection for multi-player games*, IEEE Congress on Evolutionary Computation, IEEE, 1–5, 2010.

D. Dumitrescu, R. I. Lung, **T. D. Mihoc** and R. Nagy: *Fuzzy Nash-Pareto Equilibrium: Concepts and Evolutionary Detection*, Applications of Evolutionary Computation, volume 6024, C. Di Chio et all.,editors, Springer Berlin/Heidelberg, 71–79, 2010.

D. Dumitrescu, R. I. Lung and **T. D. Mihoc**: *Evolutionary Approaches to Joint Nash – Pareto Equilibria*, Nature Inspired Cooperative Strategies for Optimization (NICSO 2010), volume 284, J. González et. all, editors, Springer Berlin/Heidelberg, 233–243, 2010.

Indexed BDI:

D. Dumitrescu, R. I. Lung and **T. D. Mihoc**: *Generative relations for evolutionary equilibria detection*, Proceedings of the 11th Annual conference on Genetic and evolutionary computation, GECCO'09, 1507–1512, ACM, New York, NY, USA, 2009.

D. Dumitrescu, R. I. Lung and **T. D. Mihoc**: *Approximating and combining equilibria in non-cooperative games*, Symbolic and Numeric Algorithms for Scientific Computing (SYNASC), 2009 11th International Symposium on, 356 – 360. IEEE, 2009.

D. Dumitrescu, R. I. Lung and **T. D. Mihoc**: *Equilibria detection in electricity market games*, Knowledge Engineering: Principles and Techniques Conference (KEPT) 2009, M. Frentiu and H. F. Pop, editors, 111–114, 2009.

D. Dumitrescu, R. I. Lung, N. Gaskó and **T. D. Mihoc**: *Evolutionary Detection of Aumann equilibrium*, Genetic And Evolutionary Computation Conference (GECCO 2010), ACM, New York, USA, 827–828, 2010.

T. D. Mihoc, R. I. Lung and D. Dumitrescu: *Notes on a Fitness Solution for Nash Equilibria in Large Games*, Proceedings of CINTI 2010, IEEE press, 53–56, 2010.

M. Sărăsan, **T. D. Mihoc**, R. I. Lung and D. Dumitrescu: *Global Search and Local Ascent for Large Cournot Games*, Studia Univ. Babeş–Bolyai, Informatica, LV(4), volume LV, 85–92, December 2010.

T. D. Mihoc, R. I. Lung, N. Gaskó and D. Dumitrescu: *Nondomination in large games: Berge-Zhukovskii equilibrium*, Studia Univ. Babeş–Bolyai, Informatica, (LVI)2, 101–106, June 2011.

Contents

Chapter 1
Introduction

This chapter contains a short introduction and the problem statement. The main contributions on the field are also presented. The organization of the thesis is in final section of this part.

Computational **Game Theory** (**GT**) is extensively used in economics, social sciences, biology, engineering, computer science, and as well in philosophy. It is the attempt to capture the agents' behavior in strategic situations, in which an individual's success in making choices depends on the choices of others.

Game equilibria are the most common solutions proposed in GT. In order to provide a adequate solutions many equilibrium concepts have been developed. Probably among these equilibria the most famous one is the Nash equilibrium.

The equilibrium concepts are motivated differently, depending on the field of application, although they often overlap or coincide. Detecting game equilibria is a fundamental computational problem within non-cooperative game theory, having non-trivial connections with multi-criteria optimization.

Problem statement

The aims are to detect all game equilibria, of a certain type, and to develop other types of equilibria that model the behaviour of real players.

Computing Nash equilibrium is one of the central open problems in computational game theory due its complexity.Some classic deterministic algorithms for approximating equilibria in n-players games have been proved to be exponential.

We will consider normal form games with pure strategies in order to simplify the players' choices. If the payoff functions are semi-continuous and strongly quasi-concave, for example, then an ε- Nash equilibrium exists in pure strategies for every positive ε. The mathematical models involved in the numerical simulations respect these conditions.

In order to cope with players different rationalities the concept of strategic game must be generalized. The non-homogeneity of players behaviour must be also considered.

Contributions

The main contributions of this dissertation to the field include:

- A new concept of non-cooperative generalized game, where the players are allowed to have different types of rationality. Nash assumed that all players are selfish, pursuing their own goals, however in reality humans also have altruistic behaviour. The strategy profile is modified in order to include the players biases.

  New equilibria for the generalized games are introduced by combining existing equilibrium concepts, thus offering new solutions. Evolutionary methods to detect the new equilibria type are applied. A fitness based on non-domination is build by combining different relations of domination (such as Pareto or Nash ascendancy).

- Several new relations of domination for Nash and ε-Nash equilibrium. The new relations are used in order to detect approximations of the Nash, and ε-Nash equilibrium respectively, by guiding the evolutionary search towards solution.

  Non-uniform ε-Nash equilibrium is introduced generalizing the concept of ε-Nash equilibrium. Generative relations for the new equilibrium are establish and the solution is computed using evolutionary techniques based on non-domination.

  Investigation of the generative relations for Nash equilibria by comparison with the Pareto domination in order to attempt equilibria computation for large games, games with great number of players. Also evolutionary methods are developed in order to attempt detecting game equilibria in large games.

Chapter 2

Background and Related Work

Chapter 2 presents several basic notions from game theory such as non cooperative games in normal form, pure strategies, and strategies profiles. After these definitions, several examples of famous games are described (Oligopoly markets of Cournot and Bertrand type games, quantum games, prisoners dilemma). The solution concepts in game theory are discussed and a formal framework for generative relations is depicted.

A game consists of a set of players (agents), and each player has a set of strategies available to her as well as a payoff function.

With respect to the relationship between the players point of view GT can be divided in two major parts: cooperative game theory and non-cooperative game theory. We will consider here the **non-cooperative** game theory with solutions in **pure strategies**.

The players will also be rational, and they will have complete information on the game. This means that each player makes the best rational decision in order to achieve his/her goal (maximaze the profit for example) and that every player has complete knowledge of the other players strategies, options and payoffs.

A player's strategy space is the set of all strategies available to him. The set of strategies available to a player can be discreet (for example in Prisoners dilemma game) or continuous (like in the oligopolies of Cournot type).

A strategy profile (or simple 'a strategy') is a complete plan of action for every stage of the game, regardless if that stage actually arises in play.

The payoff function for a player is a mapping from the cross-product of players' strategy spaces to the player's set of payoffs, i.e. the payoff function of a player takes as its input a strategy profile and yields a representation of payoff as its output.

The games will be represented in normal-form as a matrix for discreet strategies sets.

Definition 1 *A finite strategic game is defined [27] as a system*

$$\Gamma = (N, S, U)$$

where:

- $N = \{1, ..., n\}$ *is a set of* n *players;*
- *for each player* $i \in N$, $S_i = \{s_{i_1}, s_{i_2}, ..., s_{i_m}\}$ *represents the set of the actions (pure strategies) available to player* i*;*
- $S = S_1 \times S_2 \times ... \times S_n$ *is the set of all possible situations of the game;*
- *an element from* S *is a strategy profile (or strategy) of the game;*
- *for each player* $i \in N$, $u_i : S \to R$ *represents the payoff function*

$$U = (u_1, u_2, ..., u_n).$$

Let s^* be a strategy profile.

Denote by (s_{ij}, s^*_{-i}) the strategy profile obtained from s^* by replacing the strategy of player i by s_{i_j} i.e.

$$(s_{ij}, s^*_{-i}) = (s^*_i, s^*_2, ..., s^*_{i-1}, s_{ij}, s^*_{i+1}, ..., s^*_n).$$

where s^*_{-i} is a strategy profile where player's i strategy has been removed.

In order to detect all equilibria for a certain game Γ using evolutionary techniques the search can be guided similar with the detection of the Pareto set for a multi objective optimisation problem. There are many similarities between the multi objective optimisation problems and solving games. Recently an evolutionary technique has been developed for Nash equilibria detection.

We will exemplify the similarities between the two evolutionary techniques and a possible framework to detect and define more equilibria types.

A particularity of the games is, if we look from a multi objective problem point of view, that the number of players equals the number of variables and the number of objectives.

Chapter 3

Game Equilibria

Chapter 3 describes several game equilibria and the corresponding generative relations for them. The concept of non-cooperative game is generalized by taking in account the players' rationalities. A new generative relation for ε-Nash and non-uniform ε-Nash is introduced. Also by combining Nash and Pareto rationalities (selfish and altruistic behaviour) a new equilibrium is defined – Joint Nash-Pareto equilibria.

Definition 2 *Profile strategy s^* is a Nash equilibrium if the inequality $u_i(s^*) \geq u_i(s_i, s^*_{-i})$ holds for every action s_i of player i, $s_i \in S_i$.*

Remark 1 *In a pure strategy Nash equilibrium each decision-maker plays a pure, non necessarily dominant strategy, that is the best response to the strategies of other players.*

Let $k(s', s'')$ denotes the number of individual strategies from s' which replaced in s'' give better payoff for the corresponding player

$$k(s'', s') = card\{i \in \{1, ..., n\} | u_i(s'_i, s''_{-i})$$

$$> u_i(s''), s'_i \neq s''_i\}.$$

Otherwise stated $k(s'', s')$ is the number of players benefiting by switching from s'' to s' and measures the sensitivity of s'' with respect to perturbations supplied from s'. The lower sensitivity, the higher is the stability of s'' with respect to s'.

We may use

$$m(s'', s') = n - k(s'', s')$$

as a measure for the relative quality of s'' with respect to s'.

Let us consider a relation R_N on $S \times S$:

$$(s', s'') \in R_N$$

if and only if s' is better than s'' with respect to m, i.e.

$$m(s', s'') > m(s'', s').$$

Therefore $(s', s'') \in R_N$ if and only if $k(s', s'') < k(s'', s')$.

Proposition 1 *R_N is the generative relation of the Nash equilibrium, i.e. non-dominated strategies with respect to R_N are the Nash equilibria of the game.*

A solution concept that reflects the idea that players might not care about changing their strategies to a best response when the amount of utility that they could gain by doing so is very small leads to the idea of an ε-Nash equilibrium.

Definition 3 *The profile strategy s^* is a ε-Nash equilibrium if the inequality*

$$u_i(s^*) \geq u_i(s_i, s^*_{-i}) + \varepsilon$$

holds for every action s_i of player i, $s_i \in S_i$.

In an n-player game is natural to assume that players have different dispositions towards the accepted risks and possible gains. There are several ways to express the players particularities. In order to describe players different interests each player can be characterized by a particular value of ε. This represents a generalization of the standard ε-Nash equilibrium. We called it non-uniform ε-Nash equilibrium.

The concept of ε-Nash equilibrium may be generalized by considering different ε for each player respectively. This generalization is useful in order to cope with real situations.

Let us consider a vector

$$\varepsilon = (\varepsilon_1, \varepsilon_2, ..., \varepsilon_n),$$

$\varepsilon_i > 0$ represents a perturbation associated to the player i.

Definition 4 *The strategy $s^* \in S$ is a non-uniform ε-Nash equilibrium if the inequality*

$$u_i(s^*) \geq u_i(s_i, s^*_{-i}) + \varepsilon_i$$

holds for every action s_i of player i, $s_i \in S_i$, $i = \{1, 2, ..., n\}$.

Generalized Games

A natural question is what happens if the players compete against each other relying on different types of rationality. A generalized game were agents are not uniform with respect to the rationality type is introduced. The type of rationality may be considered as reflecting the player interests, bias or subjectivity. For instance the players can be more or less cooperative, more or less competitive. In this way we can also allow players to be biased toward selfish or altruistic behavior.

We assume the rationality type is described by an adequate meta-strategy concept. In a game players may assume different meta-strategies. The new paradigm offers a more realistic view and opens the possibility to further development in the Game Theory and significant applications. For instance multi-agent systems could benefit from the new approach.

The concept of generalized game with players characterized by several types of rationality is investigated. The new concepts are exemplified by considering a game were some players are Nash - and the other are Pareto-driven. An evolutionary technique for detecting the corresponding equilibrium for the generalized game is proposed.

A meta-strategy is a system

$$(s_1|r_1, s_2|r_2, ..., s_n|r_n),$$

where $(s_1, ..., s_n)$ is a strategy profile. A finite strategic generalized game is defined as a system by $G = ((N, M, U)$ where:

- N represents the set of players, $N = 1, ..., n$, n is the number of players;

- for each player $i \in N$, M_i represents the set of available meta-strategies,

- $M = M_1 \times M_2 \times ... \times M_N$ is the set of all possible situations of the generalized game and $S = S_1 \times S_2 \times ... \times S_N$ is the set of all strategies;

- for each player $i \in N$, $u_i : S \to \mathbf{R}$ represents the payoff function

$$U = (u_1, u_2, ..., u_n).$$

N-P - efficiency relation

Let us consider two meta-strategies

$$x = (x_1|r_1, x_2|r_2, ..., x_n|r_n),$$

and

$$y = (y_1|r_1, y_2|r_2, ..., y_n|r_n).$$

Let us denote by I_N the set of Nash biased players (N-players) and by I_P the set of Pareto biased players (P-players). Therefore we have

$$I_N = \{i \in \{1, ..., n\} | r_i = \text{Nash}\},$$

and

$$I_P = \{j \in \{1, ..., n\} | r_j = \text{Pareto}\}.$$

Let us introduce an operator E, measuring the relative efficiency of meta-strategies:

$$E : M \times M \to \mathbf{N},$$

defined as

$$E(x, y) = card(\{i \in I_N | u_i(x_i, y_{-i}) \geq u_i(y), x_i \neq y_i\} \cup \{j \in I_P | u_j(x) < u_j(y), x \neq y\}).$$

Remark 2 *$E(x, y)$ measures the relative efficiency of the meta-strategy x with respect to the meta-strategy y.*

The relative efficiency enables us to define a relation between meta-strategies.

Definition 5 *Let* $M_1, M_2 \in M$. *The meta-strategy* M_1 *is more efficient than meta-strategy* M_2, *and we write* $M_1 \ < E \ M_2$, *if and only if*

$$E(M_1, M_2) < E(M_2, M_1).$$

Chapter 4

Evolutionary equilibria detection

Chapter 4 proposes different evolutionary techniques to detect fast a good approximation of game equilibria. Several generative relations are proposed, relations that intend to improve the ones already developed, among them: differential generative relation for Nash equilibria, joint N-P differential generative relation, Probabilistic Nash ascendency. The relations are analysed using a coefficient of relative domination in order to examine the possible use of these relations for large games.

Let R be the generative relation for a the specific equilibrium E.

A sequence of approximations of equilibria set E may be constructed using selection methods based on generative relation R and variation operators.

A population of strategies is evolved. A population member is an n-dimensional vector representing a strategy $s \in S$. The initial population is randomly generated. Strategy population at iteration t may be regarded as the current equilibrium approximation. Subsequent application of the such operators (like the simulated binary crossover (SBX) and real polynomial mutation) is guided by a specific selection operator induced by the generative relation.

Selection for survival can be done by using a procedure based on the same selection operator or another one, also correlated to the generative relation.

In this way successive populations produce new approximations of the equilibrium front, which hopefully are better than the previous one.

It important to note that the proposed method allows to obtain an approximation of certain equilibrium also for games that do not have such an equilibrium.

The previous approach can be summarized in a technique called Relational Evolutionary Equilibria Detection (REED) as described below.

REED technique

1: Set $t = 0$;
2: Randomly initialize a population $P(0)$ of strategies;
3: **repeat**
4: Binary tournament selection and recombination using the simulated binary crossover (SBX) operator for $P(t) \to Q$;
5: Mutation on Q using real polynomial mutation $\to P$;
6: Compute the rank of each population member in $P(t) \cup P$ with respect to the generative relation. Order by rank $(P(t) \cup P)$;
7: Rank based selection for survival → P (t + 1);
8: **until** the maximum generation number is reached

Algorithm 1: Relational Evolutionary Equilibria Detection

Let us consider a Cournot duopoly.

Suppose there are two companies, that manufacture the same product in quantities q_1 and q_2 respectively. Each one's cost function is $C_i(q_i) = cq_i$ for all q_i.

Let us consider the function

$$P(Q) = \begin{cases} a - Q & \text{, if } Q \leq a \\ 0 & \text{, otherwise} \end{cases}$$

The parameters a and c are experimental determined from direct comparison between the model and the real market. We may consider in the following that $a = 24$ and $c = 9$.

The i firm profit is:

$$\begin{aligned} \pi_i(q_i, q_j) &= q_i P(Q) - C_i(q_i) \\ &= q_i \left[a - (q_i + q_j) - c \right]. \end{aligned}$$

The Nash equilibrium for this game is:

$$q^* = (q_1^*, q_2^*) = (\frac{1}{3}(a - c), \frac{1}{3}(a - c)).$$

The results for joint equilibria for diffrent rationalities are represented together in Figure 4.1.

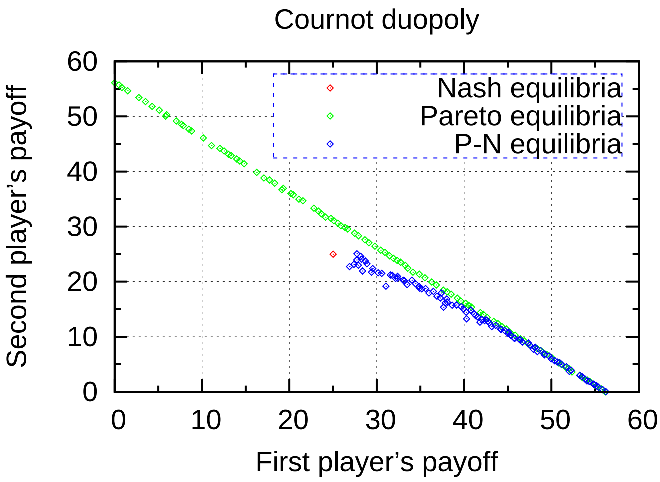

Figure 4.1: Discrete representation in the payoff space of the pure and joint equilibria detected using REED technique in 3 different runs, one for each equilibrium type respectively.

Differential generative relation of Nash equilibria

A new generative relation for Nash equilibrium is presented [18]. This relation relies on the payoff difference between perturbed and non perturbed strategies.

We introduce the measure

$$m(y, x) = \sum_{i \in N} (u_i(x_i, y_{-i}) - u_i(y)).$$

Definition 6 *The strategy x dominates y, and we write $x <_{DGN} y$, if the inequality*

$$m(x, y) < m(y, x),$$

holds.

Several numerical experiments have been performed for this game using REED technique.

We use a symmetric Cournot model with parameters $a = 24$ and $c_1 = c_2 = c_3 = 9$.

According to the results, in less than 30 generations, the algorithm converges to the Nash equilibrium point (14.00, 14.00, 14.00) for each relation. We observe that the differential Nash domination provides more accurate results than the Nash ascendency. We must consider however the particular nature of this game. For other types of games a normalisation of the deviations must be done in order to sum them.

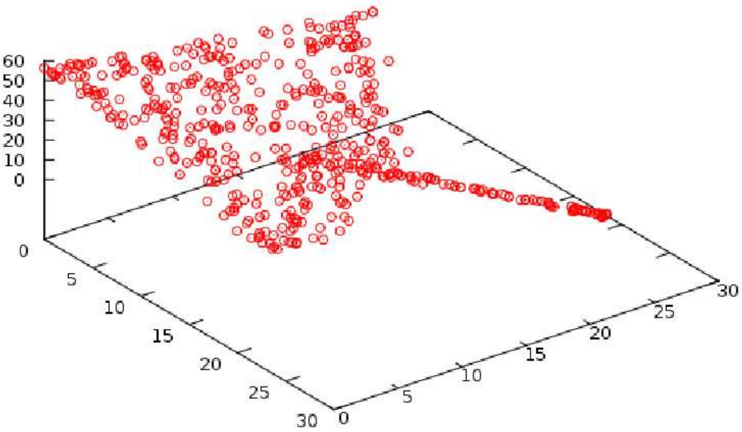

Figure 4.2: The payoffs for the Nash-Nash-Pareto front detected in less than 30 iterations for the symmetric Cournot game with the Nash–Pareto generative relation

The resulting front in the Nash-Nash-Pareto case spreads from the standard Nash equilibrium corresponding to the two player–Cournot game (25.00, 25.00) to the Nash equilibrium corresponding to the three player–Cournot game, and from there to the edges of Pareto front for the Nash–Pareto equilibria (see Figure 4.2).

Other experiments with other combinations of rationalities is presented in the thesis.

CHAPTER 5

Evolutionary techniques for equilibria detection in large games

Chapter 5 investigates the use of evolutionary techniques for large games, games that have a great number of players. Several methods are investigated, like differential evolution or memetic algorithms. A step-stone reinforcement method is developed in order to perform a local search.

Increasing the number of players is one of the challenges facing algorithmic game theory just as increasing the number of objectives is for multi-objective optimization.

Coefficient of relative dominance

Let us consider P a set of m strategy profiles,

$$R \subset S_1 \times S_2 \times ... \times S_n.$$

In order to compare Nash-ascendancy and Pareto dominance in the population P we consider a *coefficient of relative dominance*[32]

$$K_{rd} = \frac{D}{T},$$

where D denotes the number of pairs from P in which one individual dominates the other, and T the total number of unique pairs of individuals form P.

If we consider a game of Cournot type, and a random population P of strategies, and we compare K_{rd} for Pareto and Nash ascendency we obtain the results presented in Figure 5.1. For Pareto the results are similar with the ones in current literature. As the number of players increases, the chances that two individuals from P to dominate one another get extremely low.

For the Nash-ascendancy generative relation, things are quite different. As

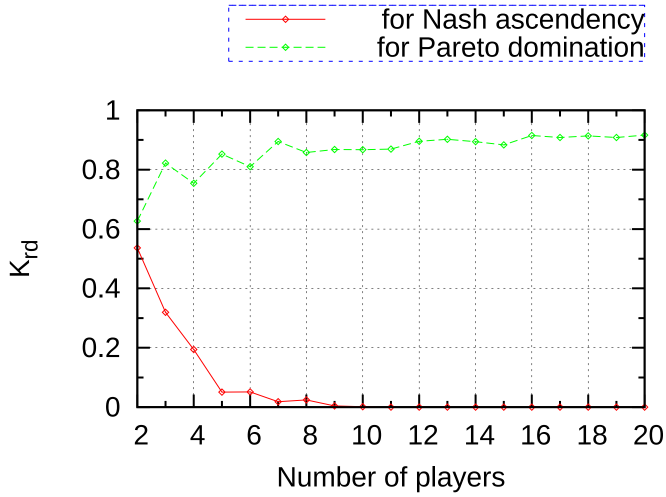

Figure 5.1: The coefficient of relative dominance computed for Nash-ascendency and for Pareto dominance respectively, for the Cournot oligopoly and for a random generated population of 50 strategies.

the number of players increases so does K_{rd} (see Figure 5.1) and the number of indifferent individuals with respect to the ascendancy relation tends to zero.

Probabilistic Nash ascendancy relation

When evaluating the Nash ascendancy relation for two strategy profiles, $2N$ payoff functions have to be computed. For a large number of players this number increases the computational complexity of the algorithm by increasing the number of fitness function evaluations [23]. One way to reduce this number is to consider only a subset of players when computing the k operator. This subset can be randomly chosen from the player and its size can be constant or it can vary.

Thus, we may consider a subset $I \in N$ composed of a percent q of randomly chosen players from N. The operator $k_q : S \times S \to N$ can be defined as

$$k_q(x, y) = |(\{i \in I | u_i(y_i, x_i) u_i(x), y_i = x_i\}|.$$

$k_q(x, y)$ counts the number of players from the set I that benefit from changing their strategies from x_i to y_i , $i \in I$ while the others keep theirs unchanged. Thus only players selected in I participate in the evaluation of the ascendancy relation reducing the number of payoff function evaluations to

$$\frac{2qN}{100}.$$

In order to evaluate the efficiency of the probabilistic generative relation we will use several evolutionary methods.

In DE (differetial evolution) algorithm, introduced by Storn and Price in 1995 [42], new candidate solutions (offspring) from a weighted difference of parent solutions.

In DE, first, all individuals are initialized and evaluated according to a given fitness function. Instead we will evaluate in the process the corresponding payoffs for each strategy profile of the game.

Afterwards as long as the termination condition is not fulfilled (e.g. the current number of fitness evaluations performed is below the maximum number of evaluations allowed) the following process will be executed: For each individual in the population, an offspring is created.

In the traditional DE, the offspring replaces the parent if it is fitter. Otherwise, the parent survives and is passed on to the next generation (iteration of the algorithm). Since on our approach the fitness is determined by non-domination with respect to the generative relation, an offspring replaces the parent only if it dominates it.

In order to detect all equilibria we used a Crowding DE (CrDE) algorithm. This method extends DE with a crowding scheme modifying the conventional DE only regarding the individual (parent) being replaced.

A *DE/rand/1/exp* scheme is used. In the form presented here, CrDE has already been used in Nash equilibria detection for large Cournot games [29].

Within CrDE individuals from population P represent strategy profiles of the game that are randomly initialized in the first generation.

As long as the final condition is not fulfilled (e.g. the current number of fitness evaluations performed is below the maximum number of evaluations allowed) for each individual i from the population, an offspring $O[i]$ is created, where $U(0, x)$ is a uniformly distributed number between 0 and x, pc denotes the probability of crossover, F is the scaling factor, and dim is the number of problem parameters (problem dimensionality).

In the traditional CrDE, the offspring $O[i]$ replaces the closest parent $P[i]$

if it is fitter. Otherwise, the parent survives and is passed on to the next generation (iteration of the algorithm).

Since on our approach the fitness is determined by non-domination with respect to the generative relation, an offspring replaces the parent only if it dominates it with respect to the proper generative relation for equilibria.

Stepping-Stone Reinforcing Search

An initial strategy profile x is randomly generated.

A mutation operator that modifies a position x_i, where i is randomly chosen with a value $\pm$ ($\pm$ is also randomly chosen) is considered. Thus $x_i' = x_i \pm \varepsilon$, where x' denotes the potential offspring.

Each step a value of i is randomly generated until an offspring Nash ascending the parent is produced using the mutation operator described above. This is equivalent to randomly searching for a player that improves its payoff when modifying its strategy with ε (either $+$ or $-$). In this case the offspring becomes the parent. This step is aimed at reinforcing the Nash 'characteristics' of the current strategy profile.

The search ends when no such i is found for the current parent, i.e. within the current strategy profile no player can improve its payoff by modifying its strategy with ε.

All operators with all variants of generative relations are tested for 10, 20, 50 and 100 players. Because the number of payoff functions is equal to the number of players, this setting creates the equivalent of four many-objective optimization problem which are known to be difficult to solve by evolutionary algorithms.

The probabilistic ascendancy relation was tested for $q = 10\%$, 30%, 50% and 100%. When $q = 100\%$ we have the Nash ascendancy relation.

The distance to Nash equilibria for the different values of q for SSRS and CrDE indicate a better performance in the case of 100 individuals for SSRS when using the probabilistic ascendancy relation. CrDE also exhibits a better performance for a higher number of player for different values of q.

Although SSRS's accuracy is better than CrDE's, its main disadvantage is that, in the current form, SSRS is only capable of detecting one NE at a time.

Memetic approaches

The above-mentioned Nash-based domination concept facilitates the comparison of two solutions, ascertaining that one is "closer" than the other to the equilibrium. Applying this domination concept in the framework of evolutionary computation, by using the generative relation within the EA comparison procedures, leads to algorithms for search and detection of a game's Nash equilibria, as the algorithm will converge to the Nash non-dominated solutions. An evolutionary model that incorporates a global search within the game solutions' space is proposed. This search is performed using a genetic algorithm that has been adapted so it detects a game's Nash equilibrium. Then, a local search algorithm is used, aimed to improve the quality (thus reducing the ”distance” to the NE) of the new population's best candidate solution.

Nash Extremal Optimization

Extremal Optimization (EO) [51, 50] is a general-purpose heuristic for finding high-quality solutions for hard optimization problems. EO has been adapted to detect NEs of noncooperative games resulting in a new method called Nash Extremal Optimization (NEO).

The main feature of EO is that the value of undesirable variables in a sub-optimal solution are replaced with new, random ones. A single candidate solution is used to search the space. Depending on the problem and the representation used this solution may be formed of several components. EO assigns a fitness to each individual component of the candidate solution and ranks them accordingly. Each iteration of the EO the component having the worst fitness is randomly and unconditionally altered; if the new solution is better then the best so found so far, it will replace it.

Within NEO the candidate solution represents a strategy profile $s \in S$, $s = (s_1, s_2, ..., s_n)$ of the game to be solved. Each component $j, j = 1, ..., n$ of strategy profile s represents the strategy of player j in that situation of the game. A natural fitness for each player i is its payoff $u_i(s)$ of which the 'worst' $u_j(s)$ is identified:

$$u_j(s) \leq u_i(s), \forall i \in \{1, ...n\}, i \neq j.$$

The only measure of quality on s is provided by the ranking of the player j, implying that all the other players are gaining more than this player for this state of the game.

Aside from this ranking there are no other parameters to adjust for selecting better solutions. The strategy of player j is randomly altered irrespective the new strategy is better or not than the previous one. Thus the near - optimal solutions are enforced by the bias against the worst solutions of the game.

Using the generative relation for Nash equilibria the new profile strategy s' is compared with the best candidate for a solution found so far s_{best}. If s' dominates s_{best}, with respect to the Nash ascendancy relation, s' becomes the new best candidate, and the search continues until a termination condition is met.

However, within NEO, as well as in EOs, the worst solution may be blocked (no further improvement is possible). This could represent a problem as the step always selects only the worst solution/player to be altered. If this happens the search process may stop without reaching even a local optimum.

Numerical experiments aim at illustrating how NEs can be computed by means of evolutionary algorithms based on appropriate generative relations for the large Cournot oligopoly model [5].

The operators are tested for 10, 50, 100, 250, 500, 750, and 1000 players. Because the number of payoff functions is equal to the number of players, this setting creates the equivalent of seven many-objective optimization problem which are known to be difficult to solve by evolutionary algorithms.

Chapter 6

Conclusions and Further Work

Chapter 6 summarizes the content of the thesis and some conclusions and further possible developments are presented.

Different equilibria, considered solutions in GT, can be characterised by generative relations between game strategy profiles. Binary generative relations for Nash and ε-Nash equilibria are considered.

An evolutionary technique (REED) based on non domination, similar with the NSGA, for detecting approximations of non cooperative game equilibria is developed. The method is validated trough comparison with the analytical results for some well known games. Cournot and Bertrand models are used to exemplify the detection of (ε-) Nash, (ε-) Pareto.

The use of generative relations allows equilibria hybridization. Each equilibrium is characterized by a particular generative relation. In this way new types of equilibria can be easily defined.

The concept of game is generalized attaching to each player his rationality. For example combining selfish players (Nash) and altruist players (Pareto) a new equilibrium concept is developed: Joint N-P equilibria.

Generative relations for Nash equilibrium based on differences between perturbations are presented. Generative relations between meta strategies induce corresponding solutions concepts named Joint differential Nash–Pareto equilibrium.

Using again REED evolutionary technique an approximation of the new defined equilibrium is detected.

Proposed method allowed to visualize the shape of equilibrium region and a qualitative study of equilibria can be accomplished. This approach is a first step toward a synthesis between computational game theory and evolutionary games.

Further work will address more equilibria types and also the detection of equilibria in mixed strategy with the use of generative relations. Designing specific algorithms for equilibria detection are other different possibilities of further development.

Some properties of the generative relation for Nash equilibria in large game are presented. As the number of players increases, the number of strategy profiles indifferent to each other with respect to Nash ascendency relation decreases, unlike the case of Pareto dominance.

The intransitive property of Nash equilibria is also outlined. All these aspects rise different challenges from those in many-criteria optimization, based on Pareto dominance.

While the Pareto dominance relation becomes useless in many-objective optimization due to too many indifferent individuals, it may be that the use of the Nash ascendancy relation would become problematic because the lack of indifferent individuals. In both cases - for many objectives/players - both relations fail to indicate efficient solutions.

The study of these properties may be useful in improving the results of evolutionary search operators designed for solving large games.

The use of a probabilistic generative relation versus the deterministic one for Nash equilibria detection in non cooperative games is studied for several Cournot oligopoly models. The probabilistic relation is introduced in order to reduce the computational complexity of the search.

Two methods, a Crowding based Differential Evolution algorithm and a Stepping Stone Reinforcing Search algorithm are used for numerical experiments. Both proposed methods, when using the probabilistic ascendancy relation are able to cope with higher number of payoffs (100) better than with less (10). This indicates the potential of the proposed approach in surpassing the problem of solving games with large number of players.

Further work includes a hybridization method for the two algorithms in order to enhance their capabilities and solve multi-player games presenting a set of multiple Nash equilibria.

Another hybrid method, called Global Search and Local Ascent algorithm is presented. GSLA combines a generational evolutionary algorithm with a hill climbing procedure in order to compute the Nash equilibria for a large non-cooperative game. The search is guided using a generative relation allowing the comparison of two strategy profiles within a game.

The efficiency of the method is evaluated for an oligopoly taking into account up to one hundred competing companies.

Results are compared with a modify version of the NSGA-II algorithm. For the given setting, GSLA significantly outperforms NSGA-II, suggesting a very good search potential.

Further work will consist in exploring this potential by using GSLA for equilibria detection in games characterized by the existence of multiple Nash equilibria.

Bibliography

[1] S. Bade, G. Haeringer, and L. Renou: *More strategies, more Nash equilibria*, Technical Report 15, School of Economics, University of Adelaide, 2004.

[2] T. Bäck: *Evolutionary Algorithms in Theory and Practice: Evolution Strategies, Evolutionary Programming, Genetic Algorithms*, Oxford Univ. Press, 1996.

[3] Mehmet Barlo, and Nuh Aygun Dalkiran: *Epsilon-Nash implementation*, Economics Letters, (102):36–32, 2009.

[4] Antoine Augustine Cournot. *Recherches sur les principes mathematiques de la theorie des richesses*, Paris, Hachette, 1838.

[5] A. F. Daughety: *Cournot oligopoly: characterization and applications*, edited by Andrew. F. Daughety, Cambridge University Press, 1989.

[6] C. Daskalakis, P. W. Goldberg, and C. H. Papadimitriou: *The Complexity of Computing a Nash Equilibrium*, STOC '06: Proceedings of the thirty-eighth annual ACM symposium on Theory of computing, 71-78, 2006.

[7] Boudewijn de Bruin: *Overmathematisation in game theory: pitting the nash equilibrium refinement programme against the epistemic programme*, Studies In History and Philosophy of Science Part A, 40(3):290–300, 2009.

[8] K. Deb, S. Agrawal, A. Pratab, and T. Meyarivan: *A fast and elitist multiobjective genetic algorithm: NSGA-II*, IEEE TRANSACTIONS ON EVOLUTIONARY COMPUTATION, 6(2), 2002.

[9] K. Deb, S. Agrawal, A. Pratab, and T. Meyarivan: *A fast elitist non-dominated sorting genetic algorithm for multi-objective optimization: NSGA-II* , Springer, 849-858, 2000.

[10] K. Deb and H. Beyer: *Self-adaptive genetic algorithms with simulated binary crossover*, Complex Systems, 9:431–454, 1995.

[11] D. Dumitrescu, Rodica Ioana Lung, and Tudor Dan Mihoc: *Approximating and combining equilibria in non-cooperative games*, Symbolic and Numeric Algorithms for Scientific Computing, International Symposium on, IEEE Computer Society, 356–360, 2009.

[12] D. Dumitrescu, Rodica Ioana Lung, and Tudor Dan Mihoc: *Equilibria detection in electricity market games*, Knowledge Engineering: Principles and Techniques Conference (KEPT) 2009, M. Frentiu and H. F. Pop, editors, 111–114, 2009.

[13] D. Dumitrescu, Rodica Ioana Lung, and Tudor Dan Mihoc: *Evolutionary equilibria detection in non-cooperative games*, Mario Giacobini, Anthony Brabazon, Stefano Cagnoni, Gianni Di Caro, Anikó Ekárt, Anna Esparcia-Alcázar, Muddassar Farooq, Andreas Fink, and Penousal Machado, editors, *Applications of Evolutionary Computing*, volume 5484 of *Lecture Notes in Computer Science*, 253–262. Springer Berlin / Heidelberg, 2009.

[14] D. Dumitrescu, Rodica Ioana Lung, and Tudor Dan Mihoc: *Generative relations for evolutionary equilibria detection*, Proceedings of the 11th Annual conference on Genetic and evolutionary computation, GECCO ’09, 1507–1512, New York, ACM press, 2009.

[15] D. Dumitrescu, Rodica Ioana Lung, Tudor Dan Mihoc, and R. Nagy: *Fuzzy Nash-Pareto Equilibrium: Concepts and Evolutionary Detection*, Applications of Evolutionary Computation, volume 6024, Springer Berlin / Heidelberg, 71-79, Eds: Cecilia Di Chio and Stefano Cagnoni and Carlos Cotta and Marc Ebner and Anikó Ekárt and Anna Esparcia-Alcazar and Chi-Keong Goh and Juan Merelo and Ferrante Neri and Mike Preuß and Julian Togelius and Georgios Yannakakis, 2010.

[16] D. Dumitrescu, Rodica Ioana Lung, N. Gaskó, and Tudor Dan Mihoc: *Evolutionary Detection of Aumann equilibrium*, Genetic And Evolutionary Computation Conference (GECCO 2010), ACM, New York, NY, USA, 827-828, 2010.

[17] D. Dumitrescu, Rodica Ioana Lung, Tudor Dan Mihoc: *Evolutionary Approaches to Joint Nash – Pareto Equilibria*, Nature Inspired Cooperative

Strategies for Optimization (NICSO 2010), volume 284, Springer Berlin / Heidelberg, 233-243, Eds: González, Juan, Pelta, David, Cruz, Carlos, Terrazas, Germán, and Krasnogor, Natalio, 2010.

[18] D. Dumitrescu, Rodica Ioana Lung, Tudor Dan Mihoc: *Meta-Rationality in Normal Form Games*, International Journal of Computers, Communications & Control V(5), volume V, 693-700, December 2010.

[19] M. Farina, and P. Amato: *On the Optimal Solution Definition for many-criteria Optimization Problems*, in Keller, J., Nasraoui, O., eds.: Proc. of the NAFIPS-FLINT Int'l Conf. 2002, IEEE Press, Piscataway NJ, 233–238, 2002.

[20] Isabel Grilo and Jean-François Mertens: *Cournot equilibrium without apology: Existence and the Cournot inverse demand function*, Games and Economic Behavior, 65(1):142–175, 2009.

[21] D.E. Goldberg: *Genetic Algorithms in Search Optimization and Machine Learning*, Publisher: Addison-Wesley Professional, 1989.

[22] Sergiu Hart: *Robert Aumann's Game and Economic Theory*, Scandinavian Journal of Economics, 108(2):185–211, 2006.

[23] H. Ishibuchi, N. Tsukamoto, Y. Nojima: *Evolutionary many-objective optimization: A short review*, in Proc. IEEE Congr. Evol. Comput., Hong Kong, 2424-2431, 2008.

[24] M. Köppen, R. Vicente-Garcia, B. Nickolay: *Fuzzy-Pareto- Dominance and Its Application in Evolutionary Multi-objective Optimization*, Evolutionary Multi-Criterion Optimization. Third International Conference, Guanajuato, México, March 2005, Springer. Lecture Notes in Computer Science Vol. 3410, 399–412, 2005.

[25] S.C. Kontogiannis, P.N. Panagopoulou, and P. G. Spirakis: *Polynomial algorithms for approximating Nash equilibria of bimatrix games*, Theor. Comput. Sci. 410, 17 (Apr. 2009), 1599-1606.

[26] Rodica Ioana Lung, A. S. Muresan, and D. A. Filip: *Solving multi-objective optimization problems by means of natural computing with application in finance*, Aplimat 2006, 445–452, Bratislav, 2006.

[27] R. I. Lung, and D. Dumitrescu: *Computing Nash equilibria by means of evolutionary computation*, in Int. J. of Computers, Communications & Control, vol. III, no. suppl. issue, 364-368, 2008.

[28] R. I. Lung, T. D. Mihoc, D. Dumitrescu: *Notes on Nash Equilibria Detection for Large Cournot Games*, 12th International Symposium on Symbolic and Numeric Algorithms for Scientific Computing Timisoara, Romania, 2010.

[29] R. I. Lung, T. D. Mihoc, D. Dumitrescu: *Nash equilibria detection for multi-player games*, WCCI 2010 IEEE World Congress on Computational Intelligence, 3447-3451, 2010.

[30] E. Maskin: *The theory of implementation in Nash equilibrium: A survey*, Social Goals and Social Organization, Hurwicz, L., Schmeidler, D., and Sonnenschein, H., editors, Cambridge University Press, 173-204, 1985.

[31] R. D. McKelvey and A. McLennan: *Computation of equilibria in finite games*, Handbook of Computational Economics, Amman, H. M., Kendrick, D. A., and Rust, J. editors, Elsevier, 1996.

[32] Tudor Dan Mihoc, Rodica Ioana Lung, and D. Dumitrescu: *Notes on a fitness solution for nash equilibria in large games*, Proceedings of CINTI 2010, Budapest, 53–56. IEEE press, 2010.

[33] J. F. Nash: *Non-cooperative games* Annals of Mathematics, (54):286–295, 1951.

[34] John von Neumann, and Oskar Morgenstern: *Theory of Games and Economic Behavior* Princeton University Press, 1994.

[35] Hakan Orbay: *Computing cournot equilibrium through maximization over prices*, Economics Letters, 105(1):71–73, 2009.

[36] Martin J. Osborne: *An Introduction to Game Theory*. Oxford University Press, NY Oxford, 2004.

[37] Martin J. Osborne, and A. Rubinstein: *A Course in Game Theory*, MIT Press, Cambridge, MA, 1994.

[38] C. Papadimitriou: *Algorithms, games, and the internet*, STOC '01: Proceedings of the thirty-third annual ACM symposium on Theory of computing,Hersonissos, Greece, pages 749 – 753, 2001.

[39] R. Radner: *Collusive behavior in noncooperative epsilon-equilibria of oligopolies with long but finite lives*, Journal of Economic Theory 22, 1980, 136–154.

[40] J. Rosenmuller: *On a Generalization of the Lemke–Howson Algorithm to Noncooperative N-Person Games* , SIAM J. Appl. Math. 21, 73 (1971).

[41] M. Sărăsan, T.D. Mihoc, R. I. Lung, and D. Dumitrescu: *Global Search and Local Ascent for Large Cournot Games,* STUDIA UNIV. BABES–BOLYAI, INFORMATICA LV(4), volume LV, 85-92, December 2010.

[42] R. Storn, and K. Price: *Differential evolution - a simple and efficient adaptive scheme for global optimization over continuous spaces*, Berkeley, CA, Tech. Rep. TR-95-012, 1995.

[43] Richard Swedberg: *Sociology and game theory: Contemporary and historical perspectives*, Theory and Society, (30):301–335, 2001.

[44] R. Thomsen: *Multimodal optimization using crowding-based differential evolution*, in Proceedings of the 2004 IEEE Congress on Evolutionary Computation. Portland, Oregon: IEEE Press, 20-23 June 2004, 1382-1389.

[45] Bert Willems, Ina Rumiantseva, and Hannes Weigt: *Cournot versus supply functions: What does the data tell us?*, Energy Economics, 31(1):38–47, 2009.

[46] Robert Wilson: *Computing Equilibria of N-Person Games*, SIAM Journal on Applied Mathematics, Vol. 21, No. 1 (Jul., 1971), pp. 80-87.

[47] Abderrahmane Ziad: *Pure-strategy ε-Nash equilibrium in n-person nonzero-sum discontinuous games*, Games and Economic Behavior, 20(2):238–249, 1997.

[48] V. V. Vazirani, N. Noam, T. Roughgarden, É. Tardos: *Algorithmic Game Theory*, Cambridge University Press, Cambridge, 2007.

[49] Yohei Sekiguchi1, Kiri Sakahara1 and Takashi Sato: Uniqueness of Nash equilibria in a quantum Cournot duopoly game, J. Phys. A: Math. Theor., 43, 2010.

[50] S. Boettcher, and A. G . Percus: Extremal Optimization: an Evolutionary Local-Search Algorithm, J. CoRR, vol. cs.NE/0209030, 2002.

[51] Boettcher, S. and Percus, A. G: Optimization with Extremal Dynamics, Physical Review Letters, 2001, jun, 86, 5211–5214.