Babes Bolyai University of Cluj Napoca
Faculty of Mathematics and Computer Science

Orbital Evolution of Trans-Neptunian Bodies

Ph.D Thesis
-Abstract-

OVIDIU C. FURDUI

SCIENTIFIC SUPERVISOR::
Prof. Dr. VASILE URECHE

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…in memory of my grandmothers Anna and Maria.

“

Abbreviations

CCD – (instrumentation) Charge Coupled Device;
Celestial object – a category that contains acronyms for natural objects in space;
Cis-Neptunian object – any astronomical body found within the orbit of Neptune;
Cubewano – is a Kuiper belt object (KBO) that orbits beyond Neptune;
Detached object – are a dynamical class of bodies in the outer Solar System beyond the orbit of Neptune;
Kuiper belt – is a region of the Solar System beyond the planets extending from the orbit of Neptune (at 30 AU) to approximately 55 AU from the Sun;
KBO – (celestial object) Kuiper Belt object;
MPC – (publication) Minor Planets and Comets;
RTNO – Resonant trans-Neptunian object;
SDO – (celestial object) scattered disc object;
TNO – (celestial object) Trans-Neptunian Object;
Trans-Neptunian Object –a celestial object in the Solar System that orbits the Sun at a greater distance on average than Neptune;
Neptune trojans – a celestial object which is in the same orbit as the planet Neptune ;

Physical Constants

Constant Name	Symbol	Value	Unit
Diameter of the Sun	$D_\odot$	$1392 \cdot 10^6$	m
Mass of the Sun	$M_\odot$	$1.989 \cdot 10^{30}$	kg
Rotational period of the Sun	$T_\odot$	25.38	days
Radius of Earth	R_A	$6.378 \cdot 10^6$	m
Mass of Earth	M_A	$5.976 \cdot 10^{24}$	kg
Rotational period of Earth	T_A	23.96	hours
Earth orbital period	Tropical year	365.24219879	days
Astronomical unit	AU	$1.4959787066 \cdot 10^{11}$	m
Light year	ly	$9.4605 \cdot 10^{15}$	m
Parsec	pc	$3.0857 \cdot 10^{16}$	m
Elementary charge	e	$1.60217733 \cdot 10^{-19}$	C
Gravitational constant	G, κ	$6.67259 \cdot 10^{-11}$	$m^3kg^{-1}s^{-2}$
Permittivity of the vacuum	ε_0	$8.854187 \cdot 10^{-12}$	F/m
Permeability of the vacuum	μ_0	$4\pi \cdot 10^{-7}$	H/m
Number π	π	3.14159265358979323846	
Number e	e	2.71828182845904523536	

Chapter 1

Introduction

In 1987, astronomer David Jewitt, then at MIT, became increasingly puzzled by ”the apparent emptiness of the outer Solar System.” He encouraged then-graduate student Jane Luu to aid him in his endeavour to locate another object beyond Pluto’s orbit, because, as he told her, ”If we don’t, nobody will.” Using telescopes at the Kitt Peak National Observatory in Arizona and the Cerro Tololo Inter-American Observatory in Chile, Jewitt and Luu conducted their search in much the same way as Clyde Tombaugh and Charles Kowal had, with a blink comparator. Initially, examination of each pair of plates took about eight hours, but the process was speeded up with the arrival of electronic Charge-coupled devices or CCDs, which, though their field of view was narrower, were not only more efficient at collecting light (they retained 90 percent of the light that hit them, rather than the ten percent achieved by photographs) but allowed the blinking process to be done virtually, on a computer screen. Today, CCDs form the basis for all astronomical detectors. In 1988, Jewitt moved to the Institute of Astronomy at the University of Hawaii. He was later joined by Jane Luu to work at the University of Hawaiis 2.24 m telescope at Mauna Kea. Eventually, the field of view for CCDs had increased to 1024 by 1024 pixels, which allowed searches to be conducted far more rapidly. Finally, after five years of searching, on August 30, 1992, Jewitt and Luu announced the ”Discovery of the candidate Kuiper belt object” (15760) 1992 QB1;[29]. Six months later, they discovered a second object in the region, 1993 FW.

Astronomers sometimes use alternative name Edgeworth-Kuiper belt to credit Edgeworth, and KBOs are occasionally referred to as EKOs. However, Brian Marsden claims neither deserve true credit; ”Neither Edgeworth or Kuiper wrote about anything remotely like what we are now seeing, but Fred Whipple did.” Conversely, David Jewitt comments that, ”If anything . . . ernandez most nearly deserves the credit for predicting the Kuiper Belt.” The term trans-Neptunian object (TNO) is recommended for objects in the belt by several scientific groups because the term is less controversial than all others it is not a synonym though, as TNOs include all objects orbiting the Sun at the outer edge of the solar system, not just those in the Kuiper belt.

Chapter 2

Astrophysical Aspects of Edgeworth-Kuiper Belt Objects

Since the discovery of Pluto, many have speculated that it might not be alone. The region now called the *Kuiper belt* had been hypothesized in various forms for decades. It was only in 1992 that the first direct evidence for its existence was found. The number and variety of prior speculations on the nature of the Kuiper belt have led to continued uncertainty as to who deserves credit for first proposing it.

The first astronomer to suggest the existence of a Trans-Neptunian population was Frederick C. Leonard. In 1930, soon after Pluto's discovery, he pondered whether it was "not likely that in Pluto there has come to light the first of a series of ultra-Neptunian bodies, the remaining members of which still await discovery but which are destined eventually to be detected".

Largest KBOs

Since the year 2000, a number of KBOs with diameters of between 500 and 1200 km (about half that of Pluto) have been discovered. 50000 Quaoar, a classical KBO discovered in 2002, is over 1200 km across. (136472) 2005 FY9 (nicknamed "Easterbunny") and (136108) 2003 EL61 nicknamed "Santa"), both announced on 29 July 2005, are larger still. Other objects, such as 28978 Ixion (discovered in 2001) and 20000 Varuna (discovered in 2000) measure roughly 500 km across.

Dwarf planet Eris

Eris (pronounced is the largest known dwarf planet in the Solar System and the ninthlargest body known to orbit the Sun directly. It is approximately 2,500 kilometres in diameter and 27% more massive than Pluto.

Eris was first spotted in January 2005 by a Palomar Observatory-based team led by Mike Brown, and its identity verified later that year. It is a trans-Neptunian object (TNO) native to a region of space beyond the Kuiper belt known as the scattered disc. Eris has one moon, ysnomia; recent observations have found no evidence of further satellites. The current distance from the Sun is 96.7 AU, roughly three times that of Pluto. With the exception of some comets the pair are the most distant known natural objects in the Solar System.

Dwarf planet Pluto

The discovery of these large KBOs in similar orbits to Pluto led many to conclude that, bar its elative size, Pluto was not particularly different from other members of the Kuiper belt. Not only did these objects approach Pluto in size, but many also possessed satellites, and were of similar composition (methane and carbon monoxide have been found both on Pluto and on the largest KBOs. Ceres was considered a planet before the discovery of its fellow asteroids, and, based on this precedent, many astronomers concluded that Pluto should also be reclassified. The issue was brought to a head by the discovery of Eris, an object in the scattered disc far beyond the Kuiper belt, that is now known to be 27 percent more massive than Pluto. In response, the International Astronomical Union (IAU), was forced to define a planet for the first time, and in so doing included in their definition that a planet must have ”cleared the neighbourhood around its orbit.” As Pluto shared its orbit with so many KBOs, it was deemed not to have cleared its orbit, and was thus reclassified from a planet to a member of the Kuiper belt.

Dwarf planet Makemake

(136472) Makemake, is the third-largest known dwarf planet in the Solar System and one of the two largest Kuiper belt objects (KBO) in the classical KBO population. Its diameter is roughly three-quarters that of Pluto. Makemake has no known satellites, which makes it unique among the largest KBOs. Its extremely low average temperature (about 30 K ($-243.2\circ C$)) means its surface is covered with methane, ethane, and possibly nitrogen ices.

Dwarf planet Haumea

(136108) Haumea, is a dwarf planet in the Kuiper belt. Its mass is one-third the mass of Pluto. It was discovered in 2004 by a team headed by Mike Brown of Caltech at the Palomar Observatory in the United States, and in 2005 by a team headed by J. L. Ortiz at the Sierra Nevada Observatory in Spain, though the latter claim has been contested. On September 17, 2008, it was accepted as a dwarf planet by the International Astronomical Union (IAU) and named after Haumea, the Hawaiian goddess of childbirth.

Sedna

0377 Sedna is a trans-Neptunian object and a likely dwarf planet discovered by Michael Brown (Caltech), Chad Trujillo (Gemini Observatory) and David Rabinowitz (Yale University) on November 14, 2003. It is currently 88 AU from the Sun, about three times as distant as Neptune. For most of its orbit Sedna is farther from the Sun than any other known dwarf planet candidate.

What came to be known as Sedna was discovered during a survey conducted with the Samuel Oschin telescope at Palomar Observatory near San Diego, California (USA) using Yale’s 160 megapixel Palomar Quest camera and was observed within days on telescopes from Chile, Spain, and the USA (Arizona, and Hawaii). NASA’s orbiting Spitzer Space Telescope was later pointed toward the object, putting an upper-bound on its diameter at roughly three-quarters that of Pluto (less than 1,600 km).

Satellites of KBOs

Of the four largest TNOs, three (Eris, Pluto, and 2003 EL61) possess satellites, and two have more than one. A higher percentage of the largest KBOs possess satellites than the smaller objects in the Kuiper belt, suggesting that a different formation mechanism was responsible. There are also a high number of binaries (two objects close enough in mass to be orbiting ”each other”) in the Kuiper belt. The most notable example is the Pluto-Charon binary, but it is estimated that over 1 percent of KBOs (a high percentage) exist in binaries.

Pluto has three known moons. The largest, Charon, is proportionally larger, compared to its primary, than any other satellite of a known planet or dwarf planet in the solar system. The other two moons, Nix and Hydra, are much smaller.

Characteristics

The Plutonian system is highly compact. The three known satellites orbit within the inner 3% of the region where prograde orbits would be stable.

Pluto and Charon have been called a double planet because Charon is large compared to Pluto (half its diameter and an eighth its mass) than any other moon is to a planet; indeed Charon is massive enough that, despite their proximity, Pluto orbits the system's barycenter at a point outside its surface.[3] Charon and Pluto are also tidally locked, so that they always present the same face toward each other.

Following Buie and Grundy's recent re-calculations taking into account older images, the orbits of the moons are confirmed to be circular and coplanar, with inclinations differing less than 0.4◦ and eccentricities less than 0.005. The inclinations are roughly 96◦ to the ecliptic (so technically the moons' movements are retrograde). The diagram on the right shows the view from the axis of the moons' orbits (declination 0°, right ascension 133◦), aligned with the HST diagram above it. As seen from Earth, these circular orbits appear foreshortened into ellipses depending on Pluto's position.

Resonances and formation

It is suspected that the Plutonian satellite system was created by a massive collision, similar to the "Big Whack" believed to have created the Earth's moon. In both cases it may be that the high angular momenta of the moons can only be explained by such a scenario. The nearly circular orbits of the smaller moons suggests that they were also formed in this collision, rather than being captured Kuiper Belt objects. This and their near orbital resonances with Charon suggest that they formed even closer to Pluto than they are at present, and that they migrated outward as Charon achieved its current orbit. If Hydra and Nix turn out to be tidally locked, as Charon is, that will settle the issue, as tidal forces are insufficient to damp their rotations in their present orbits. Both are a Lunar grey like Charon, which is consistent with a common origin. Their difference in color from Pluto, one of the reddest bodies in the Solar system due to the effects of sunlight on the nitrogen and methane ices of its surface, may be due to a loss of such volatiles during the impact or subsequent coalescence, leaving the surfaces of the moons dominated by water ice. Such an impact would be expected to create additional debris (more moons), but these must be relatively small to have avoided detection by Hubble. It is possible that there are also undiscovered irregular satellites, which are captured Kuiper Belt objects.

Origins

The precise origins of the Kuiper belt and its complex structure are still unclear, and astronomers are awaiting the completion of the Pan-STARRS survey telescope, which should reveal many currently unknown KBOs, to determine more about this.

The Kuiper belt is believed to consist of planetesimals; fragments from the original protoplanetary disc around the Sun that failed to fully coalesce into planets and instead formed into smaller bodies, the largest less than 3000 km in diameter.

Modern computer simulations show the Kuiper belt to have been strongly influenced by Jupiter and Neptune, and also suggest that neither Uranus nor Neptune could have formed in situ beyond Saturn, as too little primordial matter existed at that range to produce objects of such high mass. Instead, these planets are believed to have formed closer to Jupiter, but to have been flung outwards during the course of the Solar System's early evolution. Work in 1984 by Fernandez and Ip suggests that exchange of angular momentum with the scattered objects can cause the planets to drift. Eventually, the orbits shifted to the point where Jupiter and Saturn existed in an exact 2:1 resonance;

Jupiter orbited the Sun twice for every one Saturn orbit. The gravitational pull from such a resonance ultimately disrupted the orbits of Uranus and Neptune, causing them to switch places and for Neptune to travel outward into the proto-Kuiper belt, sending it into temporary chaos.[130]

Structure of KBOs

At its fullest extent, including its outlying regions, the Kuiper belt stretches from roughly 30 to 55 AU. However, the main body of the belt is generally accepted to extend from the 2:3 resonance at 39.5 AU to the 1:2 resonance at roughly 48 AU. The Kuiper belt is quite thick, with the main concentration extending as much as ten degrees outside the ecliptic plane and a more diffuse distribution of objects extending several times farther.

Overall it more resembles a torus or doughnut than a belt.[40] Its mean position is inclined to the ecliptic by 1.86 degrees.

Resonances of KBOs

When an object's orbital period is an exact ratio of Neptune's (a situation called a mean motion resonance), then it can become locked in a synchronised motion with Neptune and avoid eing perturbed away if their relative alignments are appropriate.

If, for instance, an object is in just the right kind of orbit so that it orbits the Sun two times for every three Neptune orbits, then whenever it returns to its original position, Neptune will always be half an orbit away from it, since it will have completed 11/2 orbits in the same time. This is known as the 2:3 (or 3:2) resonance, and it corresponds to a characteristic semi-major axis of □ 39.4 AU. This 2:3 resonance is populated by about 200 known objects, including Pluto together with its moons. In recognition of this, the other members of this family are known as Plutinos. Many Plutinos, including Pluto, often have orbits which cross that of Neptune, though their resonance means they can never collide. Many others, such as 90482 Orcus and 28978 Ixion, are over half of Pluto's size. Plutinos have high orbital eccentricities, suggesting that they are not native to their current positions but were instead thrown haphazardly into their orbits by the migrating Neptune. The 1:2 resonance (whose objects complete half an orbit for each of Neptune's) corresponds to semi-major axes of 47.7AU, and is sparsely populated. Its residents are sometimes referred to as twotinos. Minor resonances also exist at 3:4, 3:5, 4:7 and 2:5. Neptune possesses a number of trojan objects, which occupy its L_4 and L_5 points; gravitationally stable regions leading and trailing it in its orbit. Neptune trojans are often described as being in a 1:1 resonance with Neptune. Neptune Trojans are remarkably stable in their orbits and are unlikely to have been captured by Neptune, but rather to have formed alongside it.

Plutinos

Plutinos is a trans-Neptunian object in 2:3 mean motion resonance with Neptune. Plutinos are named after Pluto, which follows an orbit trapped in the same resonance, with the Italian diminutive suffix -ino. The name refers only to the orbital resonance and does not imply common physical characteristics; it was invented to describe those bodies smaller than Pluto (hence the diminutive) following similar orbits. The class includes Pluto itself and its moons.

Plutinos form the inner part of the Kuiper belt and represent about a quarter of the known Kuiper Belt objects (KBOs). Plutinos are the largest class of the resonant trans-Neptunian objects. Aside from Pluto itself and Charon, the first plutino, 1993 RO, was discovered on September 16, 1993.

The largest plutinos include *Pluto, 90482 Orcus, 28978 Ixion, 38628 Huya, (35671) 1998 SN165, and 38083 Rhadamanthus*.

The gravitational influence of Pluto is usually neglected given its small mass. However, the resonance width (the range of semi-axes compatible with the resonance) is very narrow and only a few times larger than Plutos Hill sphere (gravitational influence).

Consequently, depending on the original eccentricity, some plutinos will be driven out of the resonance by interactions with Pluto. Numerical simulations suggest that plutinos with the eccentricity 10%-30% smaller or bigger than that of Pluto are not stable on timescales.

Classical Kuiper belt objects.Cubewanos

In astronomy a cubewano is a Kuiper belt object that orbits beyond Neptune and is not controlled by an orbital resonance with the giant planet. Cubewanos have semi-major axes in the 40-50 AU range and, unlike Pluto, do not cross Neptunes orbit. They are also called classical Kuiper Belt objects.

The odd name derives from the first trans-Neptunian object (TNO) found (besides Pluto and Charon), *(15760) 1992 QB1*. Later objects were called cubewanos.

Objects identified as cubewanos include:

- *(15760) 1992 QB1*
- *(136472) 2005 FY9* the largest known cubewano and one of the largest TNO
- *(136108) 2003 EL61*, notable for its elongated shape, two moons and rapid rotation (3.9h)
- *(50000) Quaoar* and *(20000) Varuna*, each considered the largest TNO at the time of discovery
- *2002 TX300, 2002 AW197, 2002 UX25.*

However, these definitions lack precision: in particular the boundary between the classical objects and the scattered disk remains blurred. A recent classification by J. L. Elliott et al uses formal criteria based on the mean orbital parameters instead. Put informally, the definition includes the objects that have never crossed the orbit of Neptune.

According to this definition, an object qualifies as a classical KBO if:

- it is not resonant
- it has the average Tisserand's parameter exceeding 3
- its average eccentricity is less than 0.2.

Introduced by the report from the Deep Ecliptic Survey, this definition appears to be adopted in the most recent literature.

Scattered disc objects (SDOs)

The scattered disc is a sparsely populated region beyond the Kuiper belt, extending as far as 100 AU and farther. Scattered disc objects (SDOs) travel in highly elliptical orbits, usually also highly inclined to the ecliptic. Most models of solar system formation show icy planetoids first forming in the Kuiper belt, while later gravitational interactions, particularly with Neptune, displaced some of them outwards into the scattered disc.

According to the Minor Planet Center, which officially catalogues all trans-Neptunian objects, a KBO, strictly speaking, is any object that orbits exclusively within the defined Kuiper belt region regardless of origin or composition. Objects found outside the belt are classed as scattered objects. However, in some scientific circles the term ”Kuiper belt object” has become synonymous with any icy planetoid native to the outer solar system believed to have been part of that initial class, even if its orbit during the bulk of solar system history has been beyond the Kuiper belt (e.g. in the scattered disk region).

They often describe scattered disc objects as ”scattered Kuiper belt objects.” Eris, the recently discovered object now known to be larger than Pluto, is often referred to as a KBO, but is technically an SDO. A consensus among astronomers as to the precise definition of the Kuiper belt has yet to be reached, and this issue remains unresolved.

Chapter 3

Perturbation Theory

The nebular hypothesis for the formation of planetary systems is nearly 250 years old (Kant) and yet observational support for the model is relatively recent. In the standard scenario, solids in the disk surrounding the protostar begin to coagulate into macroscopic objects, which accrete to kilometer sizes. When the planetesimals become massive enough for gravitational focusing, runaway accretion begins. In the oligarchic growth phase, accretion is limited by excitations in the population induced by the largest few objects. In a protoplanetary disk, these largest planetesimals can reach a few Earth masses, sufficient to trap the nebular gas, and rapid growth of gas giant(s) can ensue.

The nebular gas is cleared by the stellar wind, and the remaining planetesimals are scattered away by the giant planets.

Perturbation System in case of Direct N-Body Integration

The expression perturbed motion implies that there is an unperturbed motion. In Celestial Mechanics the unperturbed motion is the orbital motion of two spherically symmetric bodies represented by the equations of motion (2.1), the solution of which is known in terms of simple analytical functions (see section 2.1). The constant is the product of the constant of gravitation and the sum of the masses of the two bodies considered. The numerical value of thus depends on the concrete problem and on the system of units chosen.

In Celestial Mechanics one usually makes the distinction between

1. *General Perturbation Methods*, seeking the solution in terms of series of elementary integrable functions, and
2. *Special Perturbation Methods*, seeking at some stage the solution by the methods of numerical integration.

For general perturbation methods it is mandatory not to use the original equations of motion (2.1) in rectangular coordinates, but to derive differential equations for the osculating orbital elements (see section 4.3) or for functions thereof. This procedure promises to make the best possible use of the (analytically known) solution of the twobody problem (2.1), because the osculating elements are so-called first integrals of the two-body motion.

Both, general and special perturbation methods, provide approximate solutions of the equations of motion (not regarding the few special cases which could be solved in closed form). In the former case the approximation is due to the fact that the series developments have to be terminated at some point and that sometimes the convergence of the series is not well established, in the latter case it is due to the accumulation of rounding and approximation errors. Special perturbation methods may be applied directly to the initial value problem or to the transformed equations for the osculating elements.

In this Chapter the focus is on transformations of the initial value problem with the goal to make optimum use of the analytical solution of the two-body problem. In next section, a differential equation is developed for the difference vector of a perturbed and the associated unperturbed motion. The analytical developments necessary for this purpose are rather moderate, the importance is considerable in practice. In future section, we outline the method to derive the differential equations for the osculating elements starting from the original equations of motion. The perturbation term *δf* may be rather arbitrary. The resulting equations usually are referred to as the *Gaussian perturbation equations.*

Gaussian Perturbation Equations

The concept of osculating elements, assigns one set of osculating orbital elements to every epoch *t* via the position and velocity vectors *r(t)* and $\dot{r}(t)$. There is a one-to-one relationship between the osculating elements of epoch *t* and the corresponding state vector. The transformation equations between the two sets of functions are those of the two-body problem.

Perturbation System in case of Symplectic Integration

Perturbation theory is an efficient tool for investigating the dynamics of nearly integrable Hamiltonian systems. The restricted three - body problem is the prototype of a nearly-integrable mechanical system; the integrable part is given by the two-body approximation, while the perturbation is due to the gravitational influence of the other primary. A typical example is represented by the motion of an asteroid under the gravitational attraction of the Sun and Jupiter. The mass of the asteroid is so small, that one can assume that the primaries move on Keplerian orbits. The dynamics of the small body is essentially driven by the Sun and it is perturbed by Jupiter, where the Jupiter-Sun mass ratio is observed to be about 10^{-3}. The solution of the restricted three-body problem can be investigated through perturbation theories, which were developed in the 18th and 19th centuries; they are used nowadays in many contexts of Celestial Mechanics, from ephemeris computations to astrodynamics.

Perturbation theory in Celestial Mechanics is based on the implementation of a canonical transformation, which allows us to find the solution of a nearlyintegrable system within a better degree of approximation. We review classical perturbation theory, as well as in the presence of a resonance relation. We discuss also the Birkhoff normal form around equilibrium positions and around closed trajectories.

Chapter 4

Hermite scheme: 8th order

Although the standard polynomial scheme has proved itself over more than 30 years, the rapid advance in computer technology calls for a critical appraisal and search for alternative formulations. The recent design of special-purpose computers, poses a particular challenge for software developments. The essential idea is to provide a very fast evaluation of the force and its first derivative by special hardware, and these quantities are then utilized by the integration scheme which is implemented on some front-end machine, such as a standard workstation.

In present thesis I will discuss here, the sixth- and eighth-order Hermite integrators for astrophysical *N*body simulations, which use the derivatives of accelerations up to second order (*snap*) and third order (*crackle*). These schemes do not require previous values for the corrector, and require only one previous value to construct the predictor. Thus, they are fairly easy to implement. The additional cost of the calculation of the higher order derivatives is not very high. Even for the eighth-order scheme, the number of floating-point operations for force calculation is only about two times larger than that for traditional fourth-order Hermite scheme. The sixth order scheme is better than the traditional fourth order scheme for most cases. When the required accuracy is very high, the eighth-order one is the best. These high-order schemes have several practical advantages. For example, they allow a larger number of particles to be integrated in parallel than the fourth-order scheme does, resulting in higher execution efficiency in general-purpose parallel computers.

There are two different ways to construct higher-order generalization of the Hermite scheme. The first one is to use previous timesteps, in the same way as in the original Aarseth scheme. This method was described in [**?**]. The other is to use even higher derivatives directly calculated, while still using only two points in time. Of course, it is possible to combine these two methods.

To our knowledge, there have been no published work on the latter approach combined with the individual timestep algorithm. At first sight, it looks nontrivial to combine the direct calculation of the higher-order derivatives and individual timestep algorithm. We show that the combination is actually possible and that it is not much difficult compared to the original fourth-order Hermite scheme, and also we present the result of numerical experiments, and simulation discussions.

If we do not use the individual timestep algorithm, we can easily change timesteps if we use single-step integration schemes such as Runge - Kutta methods. However, Runge- Kutta schemes cannot be combined with the individual timestep algorithm, because they require the calculation of accelerations in intermediate points. In the case of two particles with different time steps, in order to integrate the particle with longer time step, we need the position of the other particle in the past. However, with usual implementation of the individual time step algorithm, such past data is not available. In principle, we could keep the past trajectory of particles. Such schemes are not yet widely used, though a sample implementation does exist in Hut & Makino(2007).

The fourth-order Aarseth scheme had been the method of choice for the time integration of gravitational *N*-body systems. However, the optimal value for the order of the integration scheme has not been known. Makino implemented the Aarseth scheme with an arbitrary order, and performed a systematic test of the accuracy. He found that the optimal choice of the order weekly depends on the required accuracy, and if the required accuracy is very high orders higher than 4 would give better results. However, he also found that the fourth-order scheme is close to optimal for practical values of required accuracy. His result, however, is for a pure individual time step algorithm, for which the calculation cost of the acceleration depends on the order of the integrator, through the calculation cost of predictors for particles other than that integrated. McMillan (1986) and later Makino (1991) introduced the so-called block step scheme, in which the time steps of particles are quantized to powers of two so that multiple particles share exactly the same time. With this block step scheme, the calculation cost of predictors becomes much smaller than that of the force calculation for any practical value of the order of the integration scheme, and therefore high-order schemes become more efficient than in the case of the original individual time step algorithm.

Another advantage of the fourth-order Hermite scheme is that it is time-symmetric, when used with the correct-to-convergence mode. This feature has been used to achieve effective time-symmetry for the integration of internal motions of binaries or nearly circular orbits of planetesimals. Also, a time-symmetric individual time step algorithm with Hermite scheme have been implemented.

The calculation cost of the Hermite scheme per time step is somewhat higher than that of the Aarseth scheme, since the jerk must be calculated as well as the acceleration. However, roughly speaking the Hermite scheme allows the time step larger than that for the Aarseth scheme by almost a factor of two, while increase in the calculation cost seems to be less than a factor of two. Thus, by switching from the fourth-order Aarseth scheme to the fourth order-Hermite scheme, effective gain in calculation speed is achieved while the calculation program becomes simpler. This combined effect is the reason why the fourth-order Hermite scheme is now widely used.

Chapter 5

Numerical integration methods of Trans-Neptunian bodies

For several thousands of years astronomers have been asked to predict positions of the Sun, the Moon, and the planets on the celestial sphere. Especially during the Babylonian and Egyptian eras, the oldest of all sciences had been essential for producing calendars used not only for agricultural demands but also for religious rituals. The ancient methods were rather descriptive in nature, nevertheless the predictions, e.g., for Solar and Lunar eclipses were quite precise (for often the astronomer's life was tied to them). Even Kepler's laws were derived from observations without an understanding of the underlying true 'nature' of the related phenomena. It is interesting to note that Kepler already mentioned a meanwhile familiar concept, a power determining the elliptic motion of the planets, a so-called 'force'. The most renowned scientific minds tried to grasp its nature by using various tool-sets, be it Galileo Galilei's experiments or Isaac Newton's first comprehensive theoretical description by introducing his law of gravitation. Of course, our understanding of the basic principles of gravity as an interplay of space time with massed particles, as proclaimed by Einstein, has grown enormously since Newton's times, although it is far from being comprehensive, as the current problem of 'dark matter' shows quite plainly. Still, the dominant influence of gravity on the motion of massed particles at least in our Solar System is very well modeled by Newton's ansatz.

In principle, one can use two different methods for treating the equations of motion:

1. **Perturbation theory**. Perturbation theory works with series expansions of the equations of motion, or their related Hamiltonian equations, often including thousands of terms, computing analytical approximations to the solutions for a whole bundle of initial conditions. The results produced are mostly retained in form of series (also Fourier series) in a continuous parameter being identified with time, so that inserting a certain date in the series immediately leads to the particles positions in space and on the sky.
2. **The method of numerical integration**. Even though the solution of the multibody gravitational problem may not be manageably representable by means of analytical functions, it is possible to follow the systems development, through calculating evolution of the system step by step, instead of trying to achieve results, that are valid for all times. This discretization procedure constitutes the main difference to analytical approaches. In contrast to perturbation theory, the solution gained is representing just a single trajectory in phase space for a whole system of equations of motion.

Explicit Runge Kutta (RK)–type integrators are among the most popular algorithms concerning numerical analysis of initial value problems. This popularity may be due to a history dating back over a century. Also the possibility of relative ease of error control. Nevertheless, their need for a relatively high number of right–hand side function evaluations and unfavorable energy conservation properties in their classic, non–symplectic forms are downsides, that will have to be taken into account, if an application to the field of Celestial Mechanics is intended.

In the field of Computational Astronomy, is found that splitting the Hamiltonian in terms of kinetic and potential energy is not the only possibility. In fact, separating the Hamiltonian into a part representing the Keplerian motion of each planet (H_{Kep}) and a second part, containing perturbation terms due to mutual interactions with other planets (*HInt*), leads to a decrease in long–term integration error proportional to $O(\epsilon\tau^2)$ instead of $O(\tau^2)$ for a second order symplectic scheme, assumed that ($H_{int} = H_{Kep}$) with ϵ denoting the planetary to stellar mass ratio.

Remark. This special separation of the Hamiltonian will lose its advantages when the condition $H_{int} = \epsilon H_{Kep}$ is violated, which will happen, when members of the system come close to each other (close encounters).

The main problem with symplectic integration algorithms is their inherent inability to adapt step-sizes during an ongoing computation. This is due to the fact, that the Hamiltonian actually solved by numerical integration H_{num} differs from the analytic Hamiltonian *Han* by an expression proportional to all orders of the step-size τ , which means that changing the step-size automatically alters the integrated Hamiltonian, and will thus destroy the algorithm's energy conserving properties.

Given the need to calculate so-called *close encounters* 1 (CE) in celestial mechanics, this situation leaves users of symplectic algorithms with the utterly displeasing possibilities of choosing a tiny step-size right from the start, which will radically increase computational resource demands and round-off errors 2, stopping calculations whenever a CE occurs, or simply ignoring CE, admitting that from this point onward the calculation has statistical significance at best. An intriguing approach to circumnavigate this dilemma has been brought up by Chambers. He combined a second-order mixed variable symplectic integrator with fixed step-size and a Bulirsch-Stoer type extrapolation algorithm, using the symplectic part plus analytical advancing via Gauss's f and g functions, while close encounters that require changes in step-size are performed by the Bulirsch-Stoer method.

In order to get an impression of the properties of the algorithms presented, I will compare the methods to analytically predicted solutions.

Since the two-body problem is the only multi-body gravitating system, that is perfectly integrable, it is an obvious choice in this respect. For testing, the system of the Sun and Jupiter has been chosen. Initial conditions for the equinox J2000 are readily available at Solar System Dynamics of JPL. All numeric calculations were performed using the Yoshida symplectic method, the Hybrid algorithm, and the author-developed Kepler package containing the Hermite 8th order. Regarding the classic Runge-Kutta integration method, the data is used only for comparison from *Cash-Karp-Runge-Kutta* method.

Conservational properties are not the only indicators for the quality of integrators. The amount of computational resources consumed during the calculation process is equally important, as any algorithm can be trimmed to produce highly accurate results. Yet, methods will become rather unappealing, when the timescales involved in gaining usable data start to surpass weeks.
Of course, comparing algorithms contained in different package environments is rather tricky and cannot be 100% sure. This is the reason, why I chose the to split the results of CPU-time measurements according to algorithm method. As the quality of the results was set to be comparable with respect to total energy conservation, one just has to time the integration. The interference of the operating system was tracked and taken into account in the following figures.

These measurements have been done for two configurations. As one can see quite clearly, the algorithms are comparable for short integration periods. As the Hybrid algorithm accomplishes a linear error growth in the same time span, it is to be considered the most efficient.

The main dividing lines between the algorithms presented are symplecticity on the one hand, and adaptive step-size control on the other. For long-time integrations, where the orientation of single orbits is not as important as the overall energetic behavior, symplectic algorithms are probably the better choice, due to their favorable energy and angular momentum conservation properties. If one is interested in short-term, high-accuracy calculations, non-symplectic methods may be more effective. As every integrator mentioned in this chapter, contains step-size control mechanisms, close encounters during calculation runs should - in theory - not pose any major problems, although this is still an ongoing field of research.

The differences between non-symplectic algorithms are basically restricted to performance issues, and directions of energy-drifts. Concerning performance, the inner-package competitions showed that the only algorithm that is too far off to be recommended is the hybrid one, simply because the ratio of step-size to right hand side function evaluations is rather low compared to its competitors.

Chapter 6

Populations of Trans-Neptunian bodies. Dynamical classes

Our observational knowledge of the trans–Neptunian population is quite recent. The first object, Pluto, was discovered in 1930, but unfortunately this discovery was not quickly followed by the detection of other trans–Neptunian objects. It was only in 1992, with the advent of CCD cameras and a lot of perseverance, that another trans–Neptunian object *1992 QB1* was found. Now, 20 years later, we know more than 1,000 trans-Neptunian objects. Of them, about 500 have been observed for at least 3 years. A times of 3 years of observations is required in order to compute their orbital elements with some confidence. In fact, the trans–Neptunian objects move very slowly, and most of their apparent motion is simply a parallaxes effect. Our knowledge of the orbital structure of the trans–Neptunian population is therefore built on these ≈ 500 objects.

Before moving to discuss the orbital structure of the trans-Neptunian population, a brief overview of the basic facts of orbital dynamics is given.

In the absence of external perturbations, the orbital motion is perfectly elliptic. Thus, the orbital elements a, e, i, ϖ, Ω are fixed, and λ moves linearly with time, with frequency given by (4.4). When a small perturbation is introduced (for instance the presence of an additional planet), two effects are produced. First, the motion of λ is no longer perfectly linear. Correspondingly, the other orbital elements have short periodic oscillations with frequencies in the order of the orbital frequencies. Second, the angles ϖ and Ω start to rotate slowly. This motion is called *precession*. Typical precession periods in the Solar System are of the order of 10,000 - 100,000 years. Correspondingly, e and i have long periodic oscillations, with periods of the order of the precession periods.

The regularity of these short and long periodic oscillations is broken when one of the following two situations occur:

1. the perturbation becomes large, for instance when there are close approaches between the body and the perturbing planet, or when the mass of the perturber is comparable to that of the Sun (as in the case of encounters of the Solar System with other stars) or
2. the perturbation becomes resonant. In either of these cases, the orbital elements a, e, i can have large non-periodic, irregular variations.

To categorize the observed trans-Neptunian bodies into the Scattered disk and Kuiper belt, one can refer to previous works on the dynamics of trans- Neptunian bodies in the framework of the current architecture of the planetary system. For the $a < 50AU$ region, one can use the results by Duncan & Levison (1995), who numerically mapped the regions of the (a, e, i) space with $32 < a < 50$AU, which can lead to a Neptune encountering orbit within 4 Gy. Because dynamics are reversible, these are also the regions that can be visited by a body after having encountered the planet. Therefore, according to the definition above, they constitute the Scattered disk. For the $a > 50$AU region, one can use the previous results of Levison & Duncan (1997), where the the evolutions of the particles that encountered Neptune have been followed for another 4 Gy time-span. Although the initial conditions did not cover all possible configurations, one can reasonably assume that these integrations cumulatively show the regions of orbital space that can be visited by bodies transported to a > 50AU by Neptune encounters. Again, according to my definition, these regions constitute the Scattered disk.

The confirmed classical belt objects have an inclination range up to at least 32° and an eccentricity range up to 0.2, significantly higher than expected from a primordial disk, even accounting for mutual gravitational stirring. The observed distributions of eccentricity and inclination in the Kuiper belt are highly biased. High eccentricity objects have closer approaches to the Sun, and thus, they become brighter and are more easily detected. Consequently, the detection bias roughly follows curves of constant q. At first sight, this bias might explain why, in the classical belt beyond a = 44AU, the eccentricity tends to increase with semi-major axis. However, the resulting (a, e) distribution is significantly steeper than a curve q = *constant*. Thus, the apparent relative under-density of objects at low eccentricity in the region 44 < a < 48 AU is likely to be a real feature of the Kuiper belt distribution.

The co-existence of a *hot* and a *cold* population in the classical belt could be caused in one of two general manners. Either a subset of an initially dynamically cold population was excited, leading to the creation of the hot classical population, or the populations are truly distinct and formed separately. One manner in which one can attempt to determine which of these scenarios is more likely is to examine the physical properties of the two classical populations. If the objects in the hot and cold populations are physically different, it is less likely that they were initially part of the same population.

The first suggestion of a physical difference between the hot and the cold classical objects came from Levison & Stern (2001), who noted that the intrinsically brightest classical belt objects (those with lowest absolute magnitudes) are preferentially found on high inclination orbits. This conclusion has been recently verified in a bias-independent manner in Tsiganis et.all (2005), with a survey for bright objects which covered 70% of the ecliptic and found many hot classical objects but few cold classical ones.

The second possible physical difference between hot and cold classical Kuiper belt objects is their colors, which relate in an unknown manner to surface composition and physical properties. Several possible correlations between orbital parameters and color were suggested by Tegler & Romanishin (2000). The issue was clarified by Trujillo & Brown (2002), who quantitatively showed that for the classical belt, the inclination is correlated with color. In essence, the low-inclination classical objects tend to be redder than higher inclination objects.

Chapter 7

Long-time dynamics study of Trans-Neptunian Bodies

The stability of Trojan type orbits around Neptune is studied. As the first part of our investigation, we present in this chapter a global view of the stability of Trojans on inclined orbits. Using the frequency analysis method based on the FFT technique, we construct high resolution dynamical maps on the plane of initial semi-major axis versus inclination. These maps show three most stable regions, with i_0 in the range of $(0^\circ, 12^\circ)$, $(22^\circ, 36^\circ)$ and $(51^\circ, 59^\circ)$ respectively, where the Trojans are most probably expected to be found. The similarity between the maps for the leading and trailing triangular Lagrange points L_4 and L_5 confirms the dynamical symmetry between these two points. By computing the power spectrum and the proper frequencies of the Trojan motion, we figure out the mechanisms that trigger chaos in the motion. The Kozai resonance found at high inclination varies the eccentricity and inclination of orbits, while the secular resonance around 44° pumps up the eccentricity. Both mechanisms lead to eccentric orbits and encounters with Uranus that introduce strong perturbation and drive the objects away from the Trojan like orbits. This explains the clearance of Trojan at high inclination and an unstable gap around 44° on the dynamical map. An empirical theory is derived from the numerical results, with which the main secular resonances are located on the initial plane. The fine structures in the dynamical maps can be explained by these secular resonances.

Since the L_5 point of Neptune is nowadays in the direction of the Galaxy center thus not suitable for asteroid observing, it is not astonishing to see all asteroids listed in the table below are around the L_4 point.

Designation	M	ω	Ω	i	e	a (AU)
2001 QR322	57.88	160.8	151.6	1.3	0.031	30.302
2004 UP10	341.28	358.5	34.8	1.4	0.028	30.212
2005 TN53	287.04	85.7	9.3	25.0	0.065	30.179
2005 TO74	268.10	302.6	169.4	5.3	0.052	30.190
2006 RJ103	238.64	27.1	120.8	8.2	0.028	30.077
2007 VL305	352.88	215.2	188.6	28.1	0.064	30.045

Observations show that there are more objects around Jupiter's L_4 than the L_5 point, and such an asymmetry between L_4 and L_5 was also reported for Neptune. The origin of this asymmetry is discussed too in this chapter.

We present our investigations on the dynamics of the inclined Trojans in this part. As mentioned above, the value of the libration center σ_c changes only slightly with inclination, and therefore we may fix its value at 60 for L_4 and -60 for L_5 when we study the dependence of stability on the orbital inclination.

Since the first Neptune Trojan was found in 2001 their number steadily increases and now we have knowledge of 11 such asteroids around the Lagrange point L_4. It is interesting to note that two of them are on highly inclined orbits. Hence we study in this chapter the orbital stability of Neptune Trojans, with special interests on the inclined orbits. We first verified the symmetry between the L_4 and L_5 points. We found that orbits around these two points have the same stability. The only difference between them is in the value of the osculating semi-major axis of orbits at the libration center in the Trojan clouds around the L_4 and L_5 points. This difference was found due to the asymmetrical selection of initial conditions. To clarify this symmetry is important, not only because some papers argued that the $L4$ and $L5$ are dynamically asymmetrical to each other, but also because a specific formation history of Trojan clouds may affect the symmetry property. If future observing confirms the symmetry or asymmetry, it will put strong constrains on the formation scenario, which is tightly related to the early dynamical evolution of the outer solar system.

We also found that the apsidal precession of Saturn are responsible for the multiple arc structures in the dynamical map. We also check how the stability of a Trojan's orbit depends on the initial eccentricity. The preliminary results show that for most inclination values, the orbit needs a small initial eccentricity to be stable.

Chapter 8

The KEPLER software package

KEPLER is a software package for simulating the evolution of Trans-Neptunian Bodies systems and analyzing the resultant data. It is a collection of programs routines (tools) linked at the level of the UNIX operating system. The tools share a common data structure and can be combined in arbitrarily complex ways to study the dynamics of bodies or star clusters.

Gravity is the weakest of all fundamental forces in physics, far weaker than electromagnetism or the so-called weak and strong interactions between subatomic particles. However, the other three forces lose out in the competition with gravity over long distances.

The weak and strong interactions both have an intrinsically short range. Electromagnetism, while being long-range like gravity, suffers from a cancellation of attraction and repulsion in bulk matter, since there tend to be as almost exactly as many positive as negative charges in any sizable piece of matter. In contrast, gravitational interactions between particles are always attractive. Therefore, the more massive a piece of matter is, the more gravitational force it exerts on its surroundings. This dominance of gravity at long distances simplifies the job of modeling the Universe. To a first approximation, it is often a good idea to neglect the other forces, and to model the objects as if they were interacting only through gravity. In many cases, we can also neglect the intrinsic dimensions of the objects, treating each object as a point in space with a given mass. All this greatly simplifies the mathematical treatment of a system of TNO. The objects we will be studying are stars, planets, small bodies and dust or stellar systems, where the stars are so close together that they will occasionally collide and in general have frequent interesting and complex interactions. Some of the stars can take on rather extremely dense forms, like white dwarfs, and some stars may even collapse to form black holes.

However, in first approximation we can treat all these different types of objects as point particles, as far as their gravitational interactions are concerned.

Introducing individual time steps was only a first step toward the development of modern N-body codes. The presence of tight binaries produced much more of an obstacle, and throughout the seventies a variety of clever mechanisms were developed in order to deal with them efficiently. For one thing, there are problems with round-off. Two bodies in a tight orbit around each other have almost the same position vector, as seen from the center of a star cluster, where we normally anchor the global coordinate system. The separation between bodies that determine their mutual forces. When we compute the separation by subtracting two almost identical spatial vectors, we are asking for numerical trouble. The solution is to introduce a local coordinate system whenever two or more bodies undergo a close interaction. This does away with the round-off problem, but it introduces a host of administrative complexities, in order to make sure that any arbitrary configuration of bodies is locally presented correctly and that the right thing happens when two or more of such local coordinate patches encounter each other. This may not happen often, but one occurrence in a long run is enough to cause an unacceptably large error if no precautions have been taken to deal properly with such a situation.

I can continue the list of tricks that have been invented to allow every larger and denser systems of bodies to be modeled correctly. We will encounter them later on, and explain them then in detail, but just to list a few, here are some of the techniques. Numerical problems with the singularity in the two-body system have been overcome by mapping two or more interacting stars from the three-dimensional Kepler problem to a four-dimensional harmonic oscillator. The total force on particles has been split into different contributions, the first from a near zone of relatively close neighbors and the second from a far zone of all other particles, with each partial force being governed with different integration time steps. Tree codes have been used to group the contributions of a number of more and more distant zones together in ever larger chunks, for efficiency.

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