UNIVERSITY OF ORADEA

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

ABSTRACT OF THE HABILITATION THESIS

TITLE:

CONTRIBUTIONS TO THE DEVELOPMENT OF THE THEORY OF DIFFERENTIAL SUBORDINATIONS AND SUPERORDINATIONS

PRESENTED BY

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Abstract

The present thesis, which contains 8 chapters, an introduction and some references, is based on 45 selected papers written by the author (as single author or as co-author) and also on results selected from two research monographies signed by the author [GIO1], [GIO2] (all mentioned at references).

Complex Analysis is a classical field of mathematics which has its origins in the XIX-th century papers, some even written earlier, great mathematicians such as Euler, Gauss, Cauchy, Riemann, Weierstrass playing an important role in the development of this field. The main topic of this field is studying complex valued analytic functions which are divided in two large classes, holomorphic and meromorphic functions but the study of non-analytic functions is also considered.

Complex Analysis is a field of mathematics where Romanian mathematicians such as D. Pompeiu, Gh. Călugăreanu, P.T. Mocanu have important contributions and it is also a part of mathematics with many applications in different fields of sciences and techniques. An important part of Complex Analysis is Geometric Function Theory. Some of the main topics of Geometric Function Theory are geometric interpretation of certain mathematical properties expressed in analytic form and also finding analytical interpretation of certain geometric properties. Geometric Function Theory begun to develop as an individual branch of Complex Analysis in the XX-the century when the first important papers in this field were published by P. Koebe [Koe], I.W. Alexander [Alx], L. Bieberbach [Bie]. In 1916 L. Bieberbach stated the famous conjecture which could not be proved until 1984 when it was surprisingly proved by Louis de Branges when some experts were rather trying to disprove it. Bieberbach's Conjecture stimulated research in this field for almost a century.

One of the directions followed was the introduction of certain classes of univalent functions, such as starlike functions, convex functions, Mocanu functions (α-convex) for which Bieberbach's Conjecture to hold.

The problem of finding conditions to add to $f'(z) \neq 0$, $\forall\ z \in D$ (which ensures local univalence) such that the global univalence of function f in domain D to be ensured, naturally appeared. Necessary and sufficient conditions for univalence are preferred. For the case when D is a disc, such conditions were proved for the first time in 1931 by Romanian mathematician Gh. Călugăreanu [Căl] who founded the School of Geometric Function Theory at Babeş-Bolyai University, later headed by P.T. Mocanu.

Necessary and sufficient conditions for univalence are usually given in the form of differential inequalities and each sufficient condition for univalence defined a certain class of univalent functions. New methods of research appeared and developed in the field of Complex Analysis such as Loewner's parametric method, integral representation method, differential subordinations method (also known as admissibile functions method) and differential superordinations method.

The method of differential subordinations was elaborated by S.S. Miller and P.T. Mocanu in [Mi-Mo1] and [Mi-Mo2]. Using this method, important results in Geometric Function Theory of complex-valued functions could be obtained but also classical results of this field could be proved more easily. Many extensions and generalizations of those classical results could also be obtained due to this theory.

The method of differential superordinations was introduced as a dual method of differential subordinations by S.S. Miller and P.T. Mocanu in [Mi-Mo9].

The scientific activity, started with the doctoral thesis "The study of certain classes of univalent functions" presented at Babeş-Bolyai University, Faculty of Mathematics and Computer Science, 2006, scientific advisor prof. univ. dr. Grigore Şt. Sălăgean, runs in the field of Geometric Functions Theory of analytic functions (univalent functions) and of non-analytic functions and concerns mainly the following themes:

1. The study of certain classes of univalent functions obtained using some differential operators and the study of certain differential subordinations and superordinations in those classes.

2. The introduction of certain classes of univalent functions which extend the class of Mocanu Functions (α-convex).

3. The study of certain new differential operators and that of certain new classes of univalent functions using them.

4. The study of certain integral operators on classes of univalent functions.

5. Elaborating the theory for the notion of strong differential subordination as an extension of the notion of differential subordination.

6. Elaborating the theory of the notion of strong differential superordination as a dual problem of the notion of strong differential subordination.

7. Obtaining some sufficient conditions for univalence using the linear strong differential subordinations.

8. Obtaining some sufficient conditions for univalence using the nonlinear strong differential subordinations.

9. The extension of the method of differential subordinations (the method of admissible functions) to the non-analytic complex valued functions contained in classes C^1 and C^2.

10. The extension of the method of differential superordinations to the non-analytic complex-valued functions contained in classes C^1 and C^2.

The main contributions related to the 10 themes contained in the present habilitation thesis are:

In the first chapter, "Second order differential subordinations", in paragraphs 1.1, 1.2, 1.3, 1.4, 1.5, the fundamental notions related to the method of differential subordinations of admissible functions method are presented. Those notions can be found in [Mi-Mo8] or in the articles cited in this thesis.

The method of differential subordinations can be formulated shortly as follows:

Let Ω and Δ be two subsets of $\mathbb{C}$. Let p be an analytic function in the unit disc U, with $p(0) = a$ and let $\psi(r, s, t; z) : \mathbb{C}^3 \times U \to \mathbb{C}$. For this method, implications of the following form are studied:

$$\psi(p(z), zp'(z), z^2p''(z); z) \subset \Omega \Rightarrow p(U) \subset \Delta. \tag{1.1.1}$$

If Ω and Δ are simply connected domains in $\mathbb{C}$, different from $\mathbb{C}$ and $a \in \Delta$, then there exist the conformal mappings $q : U \to \Delta$, $q(U) = \Delta$, $q(0) = a$ and $h : U \to \Omega$, $h(U) = \Omega$, $h(0) = \psi(a, 0, 0; 0)$ and if ψ is holomorphic in U, then (1.1.1) can be written as:

$$\psi(p(z), zp'(z), z^2p''(z); z) \prec h(z) \Rightarrow p(z) \prec q(z). \tag{1.1.2}$$

In paragraph 1.6 notions related to Briot-Bouquet differential subordinations are presented and also the most important theorems used in studying Briot-Bouquet differential subordinations.

In paragraf 1.7 the first personal contribution to the study of certain classes of univalent functions obtained using the operator $D_\lambda^n f$ introduced in [AI-O] appears. Using this operator, the class of univalent functions denoted by $R^n(\lambda, \alpha)$ is defined.

Definition 1.7.1. [Al-O] For $0 \le \alpha < 1$, $\lambda \ge 0$, $n \in \mathbb{N}$, we let $R^n(\lambda, \alpha)$ denote the class of functions $f \in A$ which satisfy the inequality

$$\operatorname{Re}[D_\lambda^{n+1} f(z)]' > \alpha, \ z \in U,$$

where

$$D_\lambda^{n+1} f(z) = (1 - \lambda) D_\lambda^n f(z) + \lambda z [D_\lambda^n f(z)]', \ z \in U$$

$D_\lambda^{n+1} f$ is called Sălăgean differential operator,

$$D^n f(z) = z + \sum_{j=2}^{\infty} j^n a_j z^j, \ z \in U.$$

The main result obtained in paperp [GIO-GhO2] is the following:

Theorem 1.7.1. [GIO-GhO2] *Let*

$$h(z) = \frac{1 + (2\alpha - 1)z}{1 + z}, \ z \in U,$$

be a convex function in U with $h(0) = 1$, $0 \le \alpha < 1$.

If $\lambda > 0$, $n \in \mathbb{N}$, $f \in A$, and verifies the differential subordination

$$D_\lambda^{n+1} f(z) \prec h(z), \ z \in U,$$

then

$$[D_\lambda^n f(z)]' \prec q(z) = 2\alpha - 1 + \frac{2(1-\alpha)}{\lambda} \cdot \frac{1}{z^{\frac{1}{\lambda}} \sigma\left(\frac{1}{\lambda}\right)}$$

where σ is given by

$$\sigma\left(\frac{1}{\lambda}\right) = \int_0^z \frac{t^{x-1}}{1+t} dt, \ z \in U. \tag{1.7.1}$$

The function q is convex and is the best dominant.

From this theorem, the following inclusion property is easily obtained:

Corollary 1.7.1. [GIO-GhO2] *We have*

$$R^{n+1}(\alpha,\lambda)\subset R^{n+1}(\delta,\lambda)$$

where

$$\delta=\delta(\lambda,\alpha)=2\alpha-1+2(1-\alpha)\frac{1}{\lambda}\sigma\left(\frac{1}{\lambda}\right),$$

and σ is given by (1.7.1).

Using the same differential operator, the class of univalent functions denoted by $S_\lambda(\alpha,\beta,n)$ is defined as follows:

Definition 1.7.2. [GIO-GhO3] If $\alpha\geq 0$, $0\leq\beta<1$, $\lambda>0$, and $n\in\mathbb{N}=\mathbb{N}^*\cup\{0\}$, denote by $S_\lambda(\alpha,\beta,n)$ the class of functions $f\in A$, which satisfy the inequality

$$\operatorname{Re}\left[\frac{\alpha}{\lambda}\cdot\frac{D_\lambda^{n+2}f(z)}{f(z)}+\frac{D_\lambda^{n+1}f(z)}{f(z)}\left(1-\frac{\alpha(1-\lambda)}{\lambda}-\alpha\cdot\frac{zf'(z)}{f(z)}\right)\right]>\beta.$$

Remark 1.7.1. a) For $\alpha=0$, $n=0$, $\beta=0$ and $\lambda=1$, we obtain

$$\operatorname{Re}\frac{zf'(z)}{f(z)}>0,\ z\in U,$$

the class of starlike functions.

b) For $\alpha=0$, $n=0$, $0\leq\beta<1$ şi $\lambda\neq 1$ we obtain

$$\operatorname{Re}\frac{zf'(z)}{f(z)}>\beta,\ z\in U,$$

the class of starlike functions of order β.

c) For $\lambda=1$, $\alpha\geq 0$, $0\leq\beta<1$, $n\in\mathbb{N}^*\cup\{0\}$, we obtain

$$\operatorname{Re}\left[\alpha\frac{D^{n+2}f(z)}{f(z)}+\frac{D^{n+1}f(z)}{f(z)}\left(1-\alpha\frac{zf'(z)}{f(z)}\right)\right]>\beta,\ z\in U,$$

the class $S(\alpha,\beta,n)$ studied in [GIO3].

Several interesting differential subordinations are studied in this class and the following property is obtained:

Corollary 1.7.2. [GIO-GhO3] *We have the following inclusion:*

$$S_\lambda(\alpha,\beta,n)\subset S_\lambda(\alpha,\delta,n)$$

where

$$\delta=2\beta-1+\frac{2(1-\beta)}{\alpha}\sigma\left(\frac{1}{\alpha}\right),$$

$$\sigma\left(\frac{1}{\alpha}\right)=\int_0^z\frac{t^{\frac{1}{\alpha}}-1}{1+t}dt,\ z\in U.$$

Definition 1.7.3. [GIO-GhO4] If $0\leq\alpha<1$, $\lambda\geq 0$, $n,m\in\mathbb{N}^*\cup\{0\}$, let $R_n^m(\lambda,\alpha)$ denote the class of functions $f\in A_n$ which satisfy the inequality

$$\operatorname{Re}[D_\lambda^m f(z)]'>\alpha,\ z\in U,$$

where

$$D_\lambda^m f(z)=(1-\lambda)D_\lambda^m f(z)+\lambda z[D_\lambda^m f(z)]',\ z\in U.$$

Remark 1.7.2. This class of univalent functions is a generalization of previously studied classes of functions as follows:

a) For $0\leq\alpha<1$, $\lambda\geq 0$, $n=1$, $m\in\mathbb{N}^*\cup\{0\}$, we obtain

$$\operatorname{Re}[D^m f(z)]'>\alpha,$$

the class studied in [AL-O].

b) For $0\leq\alpha<1$, $n=1$, $m=1$, we obtain

$$\operatorname{Re}[f'(z)+\lambda zf''(z)]>\alpha,\ z\in U,$$

the class studied by Ponnusamy in [Pon].

c) For $\alpha = 0$, $n = 1$, $m = 0$, $\lambda = 0$, we obtain

$$\mathrm{Re}\, f'(z) > 0,\ z \in U,$$

which is the well-known univalence criteria.

The important result related to this class is contained in the following theorem:

Theorem 1.7.2. [GIO-GhO4] *The set $R_n^m(\lambda, \alpha)$ is convex.*

Several interesting differential subordinations were studied in this class as it can be seen in the following theorems:

Theorem 1.7.3. [GIO-GhO4] *Let $n, m \in \mathbb{N}$, $\lambda \geq 0$, $c \in \mathbb{C}$, $\mathrm{Re}\, c > 0$ and ω a real number given by*

$$\omega = \frac{n^2 + |c+2|^2 - |n^2 - 2c - 4|}{4n\mathrm{Re}\,(c+2)}.$$

Let h be an analytic function in U, with $h(0) = 1$ and suppose that

$$\mathrm{Re}\, \frac{zh''(z)}{h'(z)} + 1 \geq \omega.$$

If $f \in S_\lambda^m(\lambda, \alpha)$ and

$$F(z) = I_c(f) = \frac{c+2}{z^{c+1}} \int_0^z t^c f(t) dt,\ \mathrm{Re}\, c > 0,$$

then

$$[D_\lambda^m f(z)]' \prec h(z),\ z \in U,$$

implies

$$[D_\lambda^m F(z)] \prec q(z),\ z \in U,$$

where q is the solution of the differential equation

$$q(z) + \frac{n}{c+2} zq'(z) = h(z),\ h(0) = 1$$

given by

$$q(z) = \frac{c+2}{nz^{(c+2)/n}} \int_0^z t^{\frac{c+2}{n} - 1} h(t) dt.$$

Moreover q is the best dominant.

If we use in this theorem the function

$$h(z) = \frac{1 + (2\alpha - 1)z}{1+z}$$

we obtain the following (interesting) result:

Corollary 1.7.3. [GIO-GhO4] *If $\alpha < 1$, $n, m \in \mathbb{N}$, $\lambda \geq 0$, $\mathrm{Re}\, c > 0$ and I_c is defined by (1.7.3), then $I_c[S_\lambda^m(\lambda, \alpha)] \subset S_\lambda^m(\lambda, \delta)$, where*

$$\delta = \min_{|z|=1} \mathrm{Re}\, q(z)$$

and

$$q(z) = \frac{c+1}{nz^{(n+2)/n}} \int_0^z t^{\frac{c+2}{n} - 1} \cdot \frac{1 + (2\alpha - 1)t}{1+t} dt$$

and this result is sharp. Moreover

$$\delta = 2\alpha - 1 + \frac{(c+2)(2 - 2\alpha)}{n} \cdot \sigma \left(\frac{c+2}{n} \right),\ \textit{where } \sigma(x) = \int_0^z \frac{t^{x-1}}{1+t} dt.$$

The class $S_n(\beta)$ was defined in paper [Tă-GIO-Şe] as follows:

Definition 1.7.4. [Tă-GIO-Şe] If $0 \leq \beta < 1$ and $n \in \mathbb{N}$, we let $S_n(\beta)$ stand for the class of functions $f \in A$, which satisfy the inequality

$$\mathrm{Re}\, (S^n f(z)) > \beta,\ z \in U,$$

where $S^n f(z)$ is Sălăgean differential operator.

The following results were obtained related to this class:

Theorem 1.7.3. [Tă-GIO-Şe] *The set $S_n(\beta)$ is convex.*

Theorem 1.7.4. [Tă-GIO-Şe] *Let q be a convex function in U, with $q(0) = 1$, and let*

$$h(z) = q(z) + \frac{1}{c+1} zq'(z),\ z \in U,$$

where c is a complex number, with $\operatorname{Re} c > -2$.

If $f \in S_n(\beta)$ and $F = I_c(f)$, where

$$F(z) = I_c(f) = \frac{c+2}{z^{c+1}} \int_0^z t^c f(t)dt, \ \operatorname{Re} c > -2$$

then $[S^n f(z)]' \prec h(z)$, $z \in U$, *implies*

$$[S^n F(z)]' \prec q(z), \ z \in U$$

and this result is sharp.

In paragraph 1.8 the differential operator denoted by D_λ^α is presented. It was introduced as a convex combination using Sălăgean and Ruscheweyh differential operators.

Definition 1.8.1. [GIO-GhO5] Let $\alpha \geq 0$, $\lambda \geq 0$. Also let D_λ^α denote the operator given by $D_\lambda^\alpha : A \to A$,

$$D_\lambda^\alpha f(z) = (1-\lambda) S^\alpha f(z) + \lambda R^\alpha f(z), \ z \in U.$$

Here S^α and R^α are Sălăgean and Ruscheweyh differential operators, respectively.

Remark 1.8.1. a) For $\lambda = 0$, $D_0^\alpha f(z) = S^\alpha f(z)$, Sălăgean operator.

b) For $\lambda = 1$, $D_1^\alpha f(z) = R^\alpha f(z)$, $z \in U$, Ruscheweyh operator.

c) For $\alpha = 0$, $D_\lambda^0 f(z) = (1-\lambda) S^0 f(z) + \lambda R^0 f(z) = (1-\lambda) f(z) + \lambda f(z) = f(z)$.

d) For $\alpha = 1$, $D_\lambda^1 f(z) = (1-\lambda) S' f(z) + \lambda R' f(z) = (1-\lambda) z f'(z) + \lambda z f'(z) = z f'(z) = R^1 f(z) = S^1 f(z)$, $z \in U$.

The important result obtained in this paper is:

Theorem 1.8.1. [GIO-GhO5] *Let* $h(z) = \dfrac{1+(2\alpha-1)z}{1+z}$ *be a convex function in U, with $h(0) = 1$, $0 \leq \beta < 1$. Suppose that $\alpha \geq 0$, $\lambda \geq 0$ and $f \in A$ satisfies the differential subordination*

$$[D_\lambda^{\alpha+1} f(z)]' + \frac{\lambda \alpha z [R^\alpha f(z)]''}{\alpha+1} \prec h(z),$$

then

$$[D_\lambda^\alpha f(z)]' \prec q(z), \ z \in U,$$

where $q(z) = 2\beta - 1 + 2(1-\beta) \cdot \dfrac{\ln(1+z)}{z}$, $z \in U$.

The function q is convex and is the best dominant.

Using this differential operator, the class $\Sigma(\alpha, \lambda, n+1)$ is defined as follows:

Definition 1.8.2. [GIO-Că-GhO] If $0 \leq \alpha < 1$, $\lambda \geq 0$ and $n \in \mathbb{N}$, let $\Sigma(\alpha, \lambda, n+1)$ denote the class of functions $f \in \Sigma$ which satisfy the inequality

$$\operatorname{Re} \left\{ [D_\lambda^{n+1} g(z)]' + \frac{\lambda z n [R^n g(z)]''}{n+1} \right\} > \alpha,$$

where

$$f(z) = \frac{1}{z} + a_0 + a_1 z + a_2 z^2 + \dots$$

$$g(z) = z^2 f(z) = z + a_0 z^2 + a_1 z^3 + \dots, \ z \in U.$$

For this class of functions, the following property was proved:

Theorem 1.8.1. [GIO-Ca-GhO] *If $0 \leq \alpha < 1$, $\lambda \geq 0$ and $n \in \mathbb{N}$, then*

$$\Sigma(\alpha, \lambda, n+1) \subset \Sigma(\delta, \lambda, n+1),$$

where $\delta = \delta(\alpha) = 2\alpha - 1 + 2(1-\alpha) \ln 2$.

Interesting differential subordinations are studied in this paper.

The results presented in paragraph 1.9 were obtained by using the following differential operator:

$$D^{n+p-1} f(z) = z^p + \sum_{k=m+p}^{\infty} \frac{(n+k-2)!}{(n-1)!(k-1)!} a_k z^k,$$

with the property

$$(n+p) D^{n+p} f(z) = z [D^{n+p-1} f(z)]' + n D^{n+p-1} f(z), \ z \in U,$$

for the functions from class $A_m(p)$. $A_m(p)$ is the class of functions of the form:

$$f(z) = z^p + \sum_{k=m+p}^{\infty} a_k z^k, \ p, m \in \mathbb{N} = \{1, 2, 3, \dots\}.$$

This operator is called Ruscheweyh operator of order $(n+p-1)$. Using this operator, interesting differential subordinations were studied in paper [GIO-GhO-Ow]. The main result of this paper is given in the following theorem:

Theorem 1.9.1. [GIO-GhO-Ow] *Let q be a convex function in U and let function h be given by*

$$h(z) = q(z) + \frac{m}{p} z q'(z), \; z \in U,$$

with $h(0) = 1$, $m \in \mathbb{N}$, $p \in \{1, 2, \ldots\}$. If $f \in A_m(p)$ and satisfies the following differential subordination

$$\frac{1}{p} \frac{[D^{n+p-1} f(z)]'}{z^{p-1}} \prec h(z),$$

then

$$\frac{D^{n+p-1} f(z)}{z^p} \prec q(z), \; z \in U,$$

and the result is sharp.

In paragraph 1.10 results obatined using the Dziok-Srivastava operator are presented. This operator is given by:

$$H_m^l(\alpha_1, \alpha_2, \ldots, \alpha_l; \beta_1, \beta_2, \ldots, \beta_m) f(z) = z + \sum_{n=2}^{\infty} \frac{(\alpha_1)_{n-1} \ldots (\alpha_l)_{n-1}}{(\beta_1)_{n-1} \ldots (\beta_m)_{n-1}} \cdot a_n \frac{z^n}{(n-1)!} \tag{1.10.1}$$

where $\alpha_i \in \mathbb{C}$, $i = 1, 2, \ldots, l$, $\beta_j \in \mathbb{C} \setminus \{0, -1, -2, \ldots\}$, $j = 1, 2, \ldots, m$, $f \in A$.

For simplicity, we write

$$H_m^l[\alpha_1] f(z) = H_m^l(\alpha_1, \alpha_2, \ldots, \alpha_l; \beta_1, \beta_2, \ldots, \beta_m) f(z). \tag{1.10.2}$$

This operator has the property:

$$\alpha_1 H_m^l[\alpha_1 + 1] f(z) = z\{H_m^l[\alpha_1] f(z)\}' + (\alpha - 1) H_m^l[\alpha_1] f(z), \; z \in U. \tag{1.10.3}$$

The main result of this paper is given in the following theorem which gives an univalence criteria expressed using the Dziok-Srivastava operator for functions $f \in A$.

Theorem 1.10.1. [GIO10] *Let $l, m \in \{0, 1, 2, \ldots\}$, $l \le m+1$, $\alpha_i \in \mathbb{C}$, $i = 1, 2, \ldots, l$, $\beta_j \in \mathbb{C} \setminus \{0, -1, -2, \ldots\}$, $j = 1, 2, \ldots, m$ and $H_m^l[\alpha_1] f(z)$, the Dziok-Srivastava differential operator given by (1.10.1).*

If $f \in A$, $\alpha_1 \ge 0$ and $\dfrac{H_m^l[\alpha_1] f(z)}{z} \neq 0$ in U, and verifies the differential subordination

$$\frac{z\{H_m^l[\alpha_1 + 1] f(z)]'}{H_m^l[\alpha_1 + 1] f(z)} + (1 - \alpha_1) \frac{H_m^l[\alpha_1 + 1] f(z)}{H_m^l[\alpha_1] f(z)} + \alpha_1 - 1 \prec h(z),$$

where

$$h(z) = \frac{1+z}{1-z} + \frac{2z}{(1-z)[1 + \gamma + (1-\gamma) z]}, \; 0 \le \gamma < 1, \; z \in U,$$

then

$$\frac{H_m^l[\alpha_1 + 1] f(z)}{H_m^l[\alpha_1] f(z)} \prec \frac{1+z}{1-z}, \; z \in U$$

and $q(z) = \dfrac{1+z}{1-z}$ is the best dominant.

Remark 1.10.1. For $m = 0$, $l = 1$, $\alpha_1 = 1$, $H_0^1[z] f(z) = f(z)$, $H_1^1[z] f(z) = z f'(z)$, the differential subordination

$$\frac{H_m^l[\alpha_1 + 1] f(z)}{H_m^l[\alpha_1] f(z)} \prec \frac{1+z}{1-z}$$

is equivalent to $\operatorname{Re} \dfrac{z f'(z)}{f(z)} > 0$, hence $f \in S^*$.

In paper [GIO-GhO7], using the same operator the following differential subordination was obtained

$$(1 - \lambda) \frac{H_m^l[\alpha_1 + 1] f(z)}{H_m^l[\alpha_1] f(z)} + \lambda \frac{[H_m^l[\alpha_1 + 1] f(z)]'}{[H_m^l[\alpha_1] f(z)]'} \prec \frac{1-z}{1+z}$$

which is equivalent to the condition

$$\operatorname{Re} \left[(1 - \lambda) \frac{z f'(z)}{f(z)} + \lambda \left(1 + \frac{z f''(z)}{f'(z)} \right) \right] > 0,$$

the condition need in order for a function $f \in A$ to belong to class M_λ (the class of Mocanu functions).

In paragraph 1.11 the study of certain differential subordinations in the right half-plane is presented, study done by using the differential inequalities:

$$\mathrm{Re}\,[A(z)zp'(z) + B(z)p^2(z) + C(z)p(z) + D(z)] > 0$$

and

$$\mathrm{Re}\,[A(z)p^2(z) - B(z)(zp'(z))^2 + C(z)zp'(z) + D(z)] > 0.$$

Remark 1.11.1. The results obtained in Theorem 1.11.1 and 1.11.2 give sufficient conditions for univalence such as starlikeness, convexity, alpha-convexity (Mocanu functions), close-to-convexity if the following functions are used:

$$p(z) = \frac{zf'(z)}{f(z)},\ p(z) = \frac{zf''(z)}{f'(z)} + 1,\ p(z) = (1-\alpha)\frac{zf'(z)}{f(z)} + \alpha\left(\frac{zf''(z)}{f'(z)} + 1\right),\ p(z) = f'(z),\ z \in U.$$

In capter 2, "Second-order differential superordination", in paragraphs 2.1, 2.2, 2.3 the basic notions of the method of differential superordinations introduced by S.S. Miller and P.T. Mocanu [Mi-Mo9] are presented. This method can be presented shortly as follows:

Let Ω and Δ be any sets in $\mathbb{C}$, let p be analytic in the unit disc U and let $\varphi(r, s, t; z) : \mathbb{C}^3\theta U \to \mathbb{C}$.

For this method, implications of the following form are studied:

$$\Omega \subset \{\varphi(p(z), zp'(z), z^2p''(z); z) :\ z \in U\} \Rightarrow \Delta \subset p(U). \tag{2.1.1}$$

If Ω and Δ are simply connected domains in $\mathbb{C}$, then there exist the conformal mappings $q : U \to \Delta$, $q(U) = \Delta$, $q(0) = p(0)$ and $h : U \to \Omega$, $h(U) = \Omega$, $h(0) = \psi(p(0), 0, 0; 0)$. Furthermore, if functions p and $\psi(p(z), zp'(z), z^2p''(z); z)$ are univalent in U, then (2.1.1) can be written as:

$$h(z) \prec \varphi(p(z), zp'(z), z^2p''(z); z) \Rightarrow q(z) \prec p(z). \tag{2.1.2}$$

My personal contributions related to the mentioned themes using the Dziok-Srivastava differential operator can be found in paragraph 2.6.

In [GIO6] first order differential superordinations were defined and several results related to them were obtained. Here is such a result:

Theorem 2.4.1. [GIO6] *Let $\Omega \subset \mathbb{C}$, $\Omega \neq \mathbb{C}$, $q \in \mathcal{H}[a, n]$, $\alpha_1 > 0$ and let $\varphi \in \Phi_n[\Omega, q]$. If $\frac{H^l_m[\alpha_1]f(z)}{z} \in Q(1)$ and $\psi\left(\frac{H[\alpha_1]f(z)}{z}, \frac{H^l_m[\alpha_1+1]f(z)}{z}; z\right)$ is univalent in U, then*

$$\Omega \subset \left\{\varphi\left(\frac{H^l_m[\alpha_1]f(z)}{z}, \frac{H^l_m[\alpha_1+1]f(z)}{z}\right)\right\}$$

implies

$$q(z) \prec \frac{H^l_m[\alpha_1]f(z)}{z},\ z \in U,$$

where $H^l_m[\alpha_1]f(z)$ is given by (1.10.2).

Using the same differential operator, differential superordinations were obtained in paper [GIO10]. Here is a result:

Theorem 2.4.2. [GIO10] *Let $l, m \in \mathbb{N}_0 = \{0, 1, 2, \ldots\}$, $l \leq m+1$, $\alpha_i \in \mathbb{C}$, $i = 1, 2, \ldots, l$, $\beta_j \in \mathbb{C} \setminus \{0, -1, -2, \ldots\}$, $j = 1, 2, \ldots, m$ and $H^l_m[\alpha_1]f(z)$ the Dziok-Srivastava linear operator.*

If $p \in \mathcal{H}[1, 1] \cap \mathbb{Q}$, $p(U) \subset D$ and

$$\frac{z\{H^l_m[\alpha_1+1]f(z)\}'}{H^l_m[\alpha_1+1]f(z)} + (1-\alpha_1)\frac{H^l_m[\alpha_1+1]f(z)}{H^l_m[\alpha_1]f(z)} + \alpha_1 - 1$$

is univalent in U, then

$$1 + z + \frac{z}{1+\gamma+z} \prec \frac{z\{H^l_m[\alpha_1+1]f(z)\}'}{H^l_m[\alpha_1]f(z)} + (1-\alpha_1)\frac{H^l_m[\alpha_1+1]f(z)}{H^l_m[\alpha_1]f(z)} + \alpha_1 - 1$$

implies

$$1 + z \prec \frac{H^l_m[\alpha_1+1]f(z)}{H^l_m[\alpha_1]f(z)},\ z \in U,\ \gamma \geq 0$$

and $q(z) = 1 + z$ is the best dominant.

In paragraph 2.5, differential superordinations obtained by using the integral operator

$$I(f)(z) = F(z) = \left[\frac{\beta+\gamma}{z^{\gamma}} \int_0^z f^{\beta}(t) t^{\gamma-1} dt \right]^{\frac{1}{\gamma}}, \tag{2.5.1}$$

are presented. The results were published in paper [GhO-GIO2], a sandwich-type result being obtained:

$$1 + Rz \prec \frac{zF'(z)}{F(z)} \prec 1 + z, \ R \in (0,1], \ z \in U.$$

The main result can be seen in this theorem:

Theorem 2.5.1. [GIO5] *Let $R \in [0,1]$ and let h be convex in U, with $h(0) = 1$, defined by*

$$h(z) = 1 + Rz + \frac{zR}{2 + Rz}, \ z \in U.$$

If $f \in A$ and $\frac{zf'(z)}{f(z)}$ is univalent, $\frac{zF'(z)}{F(z)} \in \mathcal{H}[1,1] \cap Q$, $h(z) \prec \frac{zf'(z)}{f(z)}$, then

$$q(z) = 1 + Rz \prec \frac{zF'(z)}{F(z)}, \ z \in U,$$

where F is given by (2.5.1).

The function q is the best dominant.

In paragraph 2.6 differential subordinations obtained using Ruscheweyh differential operator are presented.

Theorem 2.6.1. [GhO-GIO6] *Let $h(z) = \frac{1 + (2\alpha - 1)z}{1 + z}$, be convex in U, with $h(0) = 1$. Let $f \in A_n$ and suppose that $[D^{m+1} f(z)]'$ is univalent in U, and $[D^m f(z)]' \in \mathcal{H}[1,1] \cap Q$. If*

$$h(z) \prec [D^{m+1} f(z)]',$$

then

$$q(z) \prec [D^m f(z)]', \ z \in U,$$

where

$$q(z) = \frac{m+1}{n z^{\frac{m+1}{n}}} \int_0^z \frac{1 + (2\alpha - 1)t}{1 + t} t^{\frac{m+1}{n} - 1} dt.$$

The function q is convex and is the best subordinant.

In chapter 3, "The univalent differential and integral operators" my personal contributions related to theme 5 mentioned at the beginning are shown.

In paragraph 3.1 the integral operator $I(f_1, f_2, \ldots, f_n)$ is presented.

Definition 3.1.1. [GIO7] Let $n, m \in \mathbb{N} \cup \{0\}$, $i \in \{1, 2, 3, \ldots, m\}$, $\alpha_i \in \mathbb{C}$, $\alpha \in \mathbb{C}$, with $\operatorname{Re} \alpha > 0$, $A^m = A \times A \times \ldots \times A$. We let $I : A^m \to A$ be the integral operator given by

$$I(f_1, f_2, \ldots, f_m)(z) = F(z) \tag{3.1.1}$$

$$= \left[\alpha \int_0^z t^{\alpha - 1} \left(\frac{R^n f_1(t)}{t} \right)^{\alpha_1} \left(\frac{R^n f_2(t)}{t} \right)^{\alpha_2} \ldots \left(\frac{R^n f_m(t)}{t} \right)^{\alpha_m} dt \right]^{\frac{1}{\alpha}},$$

where $f_i \in A$, $i \in \{1, 2, 3, \ldots, m\}$, and R^n is the Ruscheweyh differential operator.

This operator is a generalization of previously introduced integral operators.

For $n = 0$, $m = 1$, $\alpha = 1$, $\alpha_1 = 1$, $\alpha_2 = \alpha_3 = \ldots = \alpha_m = 0$ and $f \in A$, we obtain Alexander integral operator introduced in 1915 in [Alx]:

$$I(z) = \int_0^z \frac{f(t)}{t} dt, \ z \in U.$$

For $n = 0$, $m = 1$, $\alpha = 1$, $\alpha_1 = \beta \in [0,1]$, $\alpha_2 = \alpha_3 = \ldots = \alpha_m = 0$ and $f \in S$ we obtain the integral operator

$$I(z) = \int_0^z \left[\frac{f(t)}{t} \right]^{\beta} dt, \ z \in U$$

introduced by Miller-Mocanu-Reade [Mi-Mo-Re3].

For $n=1$, $m=1$, $\alpha=1$, $\alpha_1=\beta\in\mathbb{C}$, $|\beta|\leq\frac{1}{4}$, $\alpha_2=\alpha_3=\ldots=\alpha_m=0$, $R^1f(z)=zf'(z)$, $z\in U$, $f\in S$, we obtain the integral operator

$$I(z)=\int_0^z [f'(t)]^{\beta}dt,\ z\in U,$$

studied in [Pa-Pe].

For $n=0$, $m\in\mathbb{N}\cup\{0\}$, $\alpha=1$, $\alpha_i>0$, $i\in\{1,2,\ldots,m\}$, we obtain the integral operator

$$I(f_1,f_2,\ldots,f_m)(z)=F(z)=\int_0^z\left[\frac{f_1(t)}{t}\right]^{\alpha_1}\cdots\left[\frac{f_m(t)}{t}\right]^{\alpha_m}dt$$

studied in [Br-Br].

For $n,m\in\mathbb{N}\cup\{0\}$, $\alpha=1$, $\alpha_i\in\mathbb{R}$, $i\in\{1,2,\ldots,m\}$ with $\alpha_i\geq0$, we obtain the integral operator studied in [GIO-GhO6],

$$I(f_1,f_2,\ldots,f_m)(z)=\int_0^z\left(\frac{R^nf_1(t)}{t}\right)^{\alpha_1}\cdots\left(\frac{R^nf_m(t)}{t}\right)^{\alpha_m}dt.$$

For $n=0$, $m=1$, $\alpha_1=1$, $\alpha_2=\ldots=\alpha_m=0$, $\alpha\in\mathbb{C}$ with $\operatorname{Re}\alpha\geq3$, we obtain the integral operator studied in [Pes1], [Pes2]

$$G_\alpha(z)=\left[\alpha\int_0^z t^{\alpha-1}\left(\frac{g(t)}{t}\right)dt\right]^{\frac{1}{\alpha}}.$$

For $n=1$, $m=1$, $\alpha\in\mathbb{C}$, $\operatorname{Re}\alpha>0$, $\alpha_1=1$, $\alpha_2=\ldots=\alpha_m=0$, we obtain the integral operator

$$F_\alpha(z)=\left[\alpha\int_0^z t^{\alpha-1}f'(t)dt\right]^{\frac{1}{\alpha}},$$

studied in [Pes2], [Pes4].

In paper [GIO7], necessary and sufficient conditions for the operator $I(f_1,f_2,\ldots,f_m)(z)$ să to be univalent, are obtained.

Theorem 3.1.1. [GIO7] *Let $\alpha\in\mathbb{C}$, with $\operatorname{Re}\alpha>0$, $f_i\in A$, $\alpha_i\in\mathbb{C}$, $i\in\{1,2,\ldots,m\}$, with $|\alpha_1|+|\alpha_2|+\ldots+|\alpha_m|\leq1$.*

If

$$\left|\frac{z[R^nf_i(z)]'}{R^nf_i(z)}-1\right|\leq1,\ z\in U,\ i\in\{1,2,\ldots,m\},$$

then $F(z)$ given by (3.1.1) belongs to the class S.

Sufficient conditions for univalence of this operator were obtained in [GIO8], [GIO9].

Theorem 3.1.2. [GIO9] *Let $n,m\in\mathbb{N}\cup\{0\}$, $\mu\geq0$, α and c be complex numbers with $\operatorname{Re}\alpha>0$, $|c|\leq1$, $c\neq-1$ and let $f_i\in A$, $a_k\in\mathbb{C}$, $k\in\{1,2,\ldots,m\}$. If*

i) $|\alpha_1|+|\alpha_2|+\ldots+|\alpha_m|\leq(1-|c|)\dfrac{|\alpha|}{1+\mu(1+|\alpha|)}$;

ii) $|R^nf_k(z)|\leq1$;

iii) $\left|\dfrac{1-|z|^{2\alpha}}{\alpha}\left[\dfrac{z^2(R^nf_k(z))'}{(R^nf_k(z))^2}-1\right]\right|\leq1$, $z\in U$, $k\in\{1,2,\ldots,m\}$,

where R^n is the Ruscheweyh differetial operator, then the function F given by (3.1.1) belongs to the class S.

The paper [GIO9] appeared in the ISI Journal of Mathematical Inequalities (2009).

In paper [Şe-GIO] univalence conditions for the operator

$$G_{\alpha,M}(z)=\left[\frac{\alpha}{M}\int_0^z t^{\frac{\alpha}{M}-1}\left[\frac{g(t)}{t}\right]^{\frac{\alpha-1}{M^2}}dt\right]^{\frac{M}{\alpha}}\tag{3.1.2}$$

were obtained by using Ahlfors, Becker and Pascu criterion.

Theorem 3.1.3. [Şe-GIO] *Let $M\geq1$, α with $\operatorname{Re}\alpha>0$ be a complex number, $\alpha\neq1$ and c be a complex number, with $|c|\leq1$, $c\neq-1$. Let the function $g\in A$, satisfy the conditions*

$$\left|\frac{g(z)}{z}\right|\leq3M-2,\ \left|\frac{z^2g'(z)}{g^2(z)}-1\right|\leq\frac{1}{3M-2},\ z\in U\ \text{and}\ |c|+\frac{3|\alpha-1|}{|\alpha|}\leq1,$$

then the function $G_{\alpha,M}$ given by (3.1.2) belongs to the class S.

Remark 3.1.1. This integral operator is a generalization of previously introduced operators studied in [Pas1], [Pas2], [Pes1], [Pes2].

In paragraph 3.2 the operators $I_p(\beta, m, n; \lambda, l)$ and L are presented and also a class denoted by $S_p^{\alpha}(\beta, m, n; \lambda, l)$.

Definition 3.2.1. [GIO23] For $l, \lambda, \beta \in \mathbb{R}$, $l \geq 0$, $\beta \geq 0$, $\lambda \geq 0$, $n, p \in \mathbb{N}$, $m \in \mathbb{N} \cup \{0\}$ and $f \in A(p, n)$, we define the multiplier transformation $I_p(\beta, m, n; \lambda, l)$ on $A(p, n)$ by the following infinite series

$$I_p(\beta, m, n; \lambda, l)f(z) = z^p + \sum_{k=p+n}^{\infty} \left[\frac{(1-\lambda)(p-k) + l + \dfrac{(m+k-1)!}{m!(k-1)!}}{p + l - k + (m+k-1)!} \right]^{\beta} \frac{(m+k-1)!}{m!k!} a_k z^k, \ z \in U. \tag{3.2.1}$$

$A(p, n)$ is the class of functions of the form

$$f(z) = z^p + \sum_{k=p+n}^{\infty} a_k z^k, \ z \in U.$$

This operator is a generalization of a great number of known differential operators.

Definition 3.2.2. [GIO23] Let $0 \leq \alpha < 1$, $m \in \mathbb{N} \cup \{0\}$, $p, n \in \mathbb{N}$, $l, \lambda, \beta \in \mathbb{R}$, $l \geq 0$, $\lambda \geq 0$, $\beta \geq 0$. A function $f \in A(p, n)$ is said to be in the class $S_p^{\alpha}(\beta, m, n; \lambda, l)$ if it satisfies the following inequality

$$\operatorname{Re} I_p'(\beta, m, n; \lambda, l)f(z) > \alpha, \ z \in U, \tag{3.2.2}$$

where $I_p(\beta, m, n; \lambda, l)f(z)$ is the operator given by (3.2.1).

Theorem 3.2.1. [GIO23] If $0 \leq \alpha < 1$, $m \in \mathbb{N} \cup \{0\}$, $n \in \mathbb{N}$, $l, \lambda, \beta \in \mathbb{R}$, $l \geq 0$, $\lambda > 0$, $\beta \geq 0$, $f \in A(1, n) = A_n$, $\dfrac{E(m, l, k)}{\lambda} > 0$, where

$$E(m, l, k) = 1 + l - k + \frac{(m+k-1)!}{m!(k-1)!} \tag{3.2.3}$$

then $S_1^{\alpha}(\beta + 1, m, n; \lambda, l) \subset S_1^{\alpha}(\delta, m, n; \lambda, l)$, unde

$$\delta = \delta(\alpha, m, n, l; k, \lambda) = 2\alpha - 1 + 2(1-\alpha)\frac{E(m, l, k)}{\lambda n}\sigma\left[\frac{E(m, l, k)}{\lambda n}\right]$$

and

$$\sigma\left[\frac{E(m, l, k)}{\lambda n}\right] = \int_0^z \frac{t^{\frac{E(m,l,k)}{\lambda n} - 1}}{1+t} dt, \ z \in U.$$

Definition 2.3.3. [GIO23] Let $0 \leq \alpha < 1$, $m \in \mathbb{N} \cup \{0\}$, $p, n \in \mathbb{N}$, $l, \lambda, \beta \in \mathbb{R}$, $\lambda \geq 1$, $\beta \geq 0$, $f \in A(p, n)$. We denote by $L : A(p, n) \to A(P, n)$ the integral operator defined by $L(f) = F$, where F is given by

$$L(f) = F(z) = \frac{p + E(m, l, k)}{\lambda z^{\frac{p(1-\lambda) + E(m, l, k) - 1}{\lambda}}} \int_0^z f(t) t^{\frac{p(1-\lambda) + E(m,l,k) - 1}{\lambda}} dt \tag{3.2.4}$$

with $E(m, l, k)$ given by (3.2.3).

Remark 3.2.1. a) For $p = 1$, $l = 0$, $m = 1$, $\lambda = 1$, $f \in A_n$

$$F(z) = \int_0^z \frac{f(t)}{t} dt$$

we obtain the Alexander operator [Alx].

b) For $p = 1$, $m = 1$, $\lambda = 1$, $l = 1$, $f \in A_n$

$$F(z) = \frac{2}{z} \int_0^z f(t) dt$$

we obtain the Libera operator [Lib].

c) For $p = 1$, $l \geq 0$, $m = 1$, $\lambda = 1$, $f \in A_n$

$$F(z) = \frac{l+1}{z^l} \int_0^z f(t) t^{l-1} dt$$

we obtain the Bernardi operator [Ber].

Using the integral operator given by (3.2.4), the next theorem is proved:

Theorem 3.2.1. [GIO23] *Let* $0 \le \alpha < 1$, $m \in \mathbb{N}\cup\{0\} p = 1$, $n \in \mathbb{N}$, $l, k, \beta \in \mathbb{R}$, $\lambda \ge 1$, $\beta \ge 0$ *and* $f \in A_n$. *Then* f *belongs to the class* $S_1^\alpha(\beta, m, n; \lambda, l)$ *if and only if* F *defined by (3.2.4) belongs to the class* $S_1^\alpha(\beta+1, m, n; \lambda, l)$.

In paragraph 3.3 the starlikeness of Bernardi operator is studied.

In papers [Le-Mi-Zl] and [Pas2] it has been proved that:

i) $L_\gamma[S^*] \subset S^*$, (ii) $L_\gamma[K] \subset K$, (iii) $L_\gamma(C) \subset C$,
where S^* is the class of starlike functions, K is the class of convex functions, C is the class of close-to-convex functions and

$$L_\gamma[f](z) = F(z) = \frac{\gamma+1}{z^\gamma}\int_0^z f(t)t^{\gamma-1}dt \tag{3.3.1}$$

is the Bernardi operator.

In paper [GIO14], this property was extended to starlike functions of negative order:

$$S\left(-\frac{1}{2\gamma}\right) = \left\{f \in A,\ \frac{f(z)}{z} \neq 0,\ \mathrm{Re}\,\frac{zf'(z)}{f(z)} > -\frac{1}{2\gamma},\ \gamma \ge 1\right\}.$$

Theorem 3.2.2. [GIO14] *Let* $f \in A$, $\dfrac{f(z)}{z} \neq 0$, $z \in U$, $\gamma \ge 1$. *If*

$$\mathrm{Re}\,\frac{zf'(z)}{f(z)} > -\frac{1}{2\gamma}$$

then $F \in S^*$, *where* F *is given by (3.3.1).*

In paper [GIO13] it has been proved that if $f \in K\left(-\dfrac{1}{2\gamma}\right)$, $\gamma \ge 1$, then F given by (3.3.1) is convex.

Theorem 3.2.3. [GIO13] *Let* $f \in A$, $\gamma \ge 1$. *If*

$$\mathrm{Re}\left[\frac{zf''(z)}{f'(z)} + 1\right] \ge -\frac{1}{2\gamma},$$

then F *given by (3.3.1) is convex.*

In paragraph 3.4, conditions for the image of a convex function f through the integral operator

$$I(f) = F(z) = \frac{1}{[g(z)]^c}\int_0^z f(w)g(w)^{c-1}g'(w)dw \tag{3.4.1}$$

to be convex, where $c \in \mathbb{C}$ with $\mathrm{Re}\,c > 0$, are shown.

Theorem 3.4.1. [GIO-GhO8] *Let* I *be the integral operator defined by*

$$I(f) = F(z) = \frac{1}{[g(z)]^c}\int_0^z f(w)g(w)^{c-1}g'(w)dw, \tag{3.4.1}$$

$z \in U$, $f, g \in \mathcal{H}(U)$, *and suppose that*

(i) $\mathrm{Re}\,\dfrac{czg'(z)}{g(z)} > 0,\ z \in U,\ \mathrm{Re}\,c > 0,$

(ii) $\mathrm{Re}\left(\dfrac{zg''(z)}{g'(z)} + 1\right) > \mathrm{Re}\,\dfrac{(c+1)zg'(z)}{g(z)},\ z \in U,$

(iii) $I(C) \subset C$.

Then $I(K) \subset K$.

In paragraph 3.5, necessary conditions such that the image of function $f \in A$ through the integral operator

$$I(f)(z) = F(z) = \frac{2}{z}\int_0^z f(t)dt \tag{3.5.1}$$

to be a convex functions, are presented.

Theorem 3.5.2. [GhO-GIO5] *Let* $M \in [0, M_0]$, *where* $M_0 = 0,41284489$ *is the positive root of the equation:*

$$7M^8 + 14M^7 + 48M^6 + 30M^5 + 67M^4 + 18M^3 + 18M^2 + 2M - 8 = 0.$$

If $f \in A$ *and satisfies the inequality*

$$\left|\frac{zf'(z)}{f(z)} - 1\right| < M$$

then $\mathrm{Re}\,\dfrac{zF'(z)}{F'(z)} + 1 > 0$, *where* F *is given by (3.5.1).*

In paragraph 3.6, conditions such that the image of a function $f \in \mathcal{H}[1,1]$ through Bernardi operator to be a convex function, are shown.

Theorem 3.6.1. [GIO21] *Let $f \in \mathcal{H}[1,1]$, $\gamma \geq 1$ and*

$$L_\gamma[f](z) = F(z) = \frac{\gamma}{z^\gamma} \int_0^z f(t)t^{\gamma-1}dt, \ z \in U. \tag{3.6.1}$$

If

$$\text{Re}\left[\frac{zf''(z)}{f'(z)} + 1\right] > -\frac{1}{2\gamma}, \ z \in U,$$

then F given by (3.6.1) is convex.

In chapter 4, "Classes of univalent functions which extend the class of Mocanu functions", several classes which extend the class of Mocanu functions (α-convex functions)

$$M_\alpha = \left\{f \in A, \ \text{Re}\left[(1-\alpha)\frac{zf'(z)}{f(z)} + \alpha\left(\frac{zf''(z)}{f'(z)} + 1\right)\right] > 0, \ z \in U\right\},$$

are presented.

The duality theorem for this class is well-known:

Theorem 4.4.1. [Mo-Bu-Să] *If $\alpha \geq 0$, then $f \in M_\alpha \Leftrightarrow F \in S^*$, where*

$$F(z) = f(z)\left[\frac{zf'(z)}{f(z)}\right]^\alpha. \tag{4.1.1}$$

In paper [GhO-GIO3], the class of functions denoted by $M_{\alpha,\beta}$ is introduced as follows:

Definition 4.1.1. [GhO-GIO3] Let $\alpha, \beta \in \mathbb{R}$ and $f \in A$ with $\dfrac{f(z)f'(z)}{z} \neq 0$ for $z \in U$. We say that the function f belongs to the class $M_{\alpha,\beta}$ if the function $F : U \to \mathbb{C}$, defined by

$$F(z) = z\left[\frac{f(z)}{z}\right]^\beta \left[\frac{zf'(z)}{f(z)}\right]^\alpha \tag{4.1.2}$$

is starlike in U.

Remark 4.1.1. a) If $\beta = 1$, $\alpha \geq 0$, then

$$F(z) = f(z)\left[\frac{zf'(z)}{f(z)}\right]^\alpha$$

and $M_{\alpha,1} = M_\alpha$, i.e. the class $M_{\alpha,1}$ coincides with the class of α-convex functions (Mocanu functions).

b) If $\alpha = \beta = 1$, $F(z) = zf'(z)$, $z \in U$ and $M_{1,1} = K$, i.e. the class $M_{1,1}$ coincides with the class of convex functions.

The important results proved for this class can be seen in the following theorems.

Theorem 4.1.1. [GhO-GIO3] *Let $\alpha \in \mathbb{R}$ and $\beta \geq 1$. Then $M_{\alpha,\beta} \subset S^*$.*

Theorem 4.1.2. [GhO-GIO3] *Let $\alpha, \beta \in \mathbb{R}$ with $\alpha \geq 0$, $\beta \geq 1$ and let $\lambda \in (0, \lambda_1)$, where*

$$\lambda_1 = \frac{2\beta - \alpha - 2 + \sqrt{(2\beta-\alpha-2)^2 + 8\alpha\beta}}{4\beta}, \ \lambda_1 \in (0,1). \tag{4.1.3}$$

If $f \in M_{\alpha,\beta}$, then $f \in S^*(\lambda)$.

In paper [GhO-GIO4] first order differential subordinations were studied and the best dominant for a function $f \in M_{\alpha,\beta}$ is given.

Theorem 4.1.3. [GhO-GIO4] *Let $\alpha, \beta, \lambda \in \mathbb{R}$, $\alpha > 0$, $\beta \geq 1$, $\lambda \in (0, \lambda_1)$, where λ_1 is given by (4.1.3). If $f \in M_{\alpha,\beta}$, then*

$$\frac{zf'(z)}{f(z)} \prec (1-\lambda)q(z) + \lambda$$

where q is given by

$$q(z) = \left[\frac{\beta(1-\lambda)}{\alpha}(1-z)^{\frac{2}{\alpha}} \int_0^1 \frac{t^{\frac{\beta}{\alpha}-1}}{(1-tz)^{\frac{2}{\alpha}}}dt\right]^{-1} - \frac{\lambda}{1-\lambda}.$$

The function q is the best dominant.

In paper [GIO11] the class $M^n_{\alpha,\beta}(\delta)$ was defined.

Theorem 4.1.4. [GIO11] *Let $\alpha, \beta, \delta \in \mathbb{R}$, $1 - \beta \leq \delta < 1$. Then $M^n_{\alpha,\beta}(\delta) \subset S^*$.*

Theorem 4.1.5. [GIO11] *Let $\alpha, \beta, \delta \in \mathbb{R}$ with $\alpha > 0$, $\beta > 0$, $0 \le \delta < 1$, let n be a positive integer and let $\lambda \in (0, \lambda_1)$, where*

$$\lambda_1 = 2\delta + 2\beta - 2 - \alpha n + \sqrt{(2\delta + 2\beta - 2 - \alpha n)^2 + 8\alpha\beta n},\ \ \lambda_1 \in (0, 1). \tag{4.1.4}$$

If $f \in M^n_{\alpha,\beta}(\delta)$ then $f \in S^*(\lambda)$.

In paper [GIO12] certain differential subordinations for functions from class $M^n_{\alpha,\beta}(\delta)$ were studied and the best dominant was given in the theorem:

Theorem 4.1.6. [GIO12] *Let n be a positive integer, $\alpha, \beta, \delta, \lambda$ be real numbers with $\alpha > 0$, $\beta \ge 1$, $0 \le \delta < 1$, $\lambda \in (0, \lambda_1)$, where*

$$\lambda_1 = \frac{2\delta + 2\beta - 2 - \alpha n + \sqrt{(2\delta + 2\beta - 2 - \alpha n)^2 + 8\alpha\beta n}}{4\beta},\ \ \lambda_1 \in [0, 1).$$

If $f \in M^n_{\alpha,\beta}(\delta)$, then

$$\frac{zf'(z)}{f(z)} \prec (1 - \lambda)q(z) + 1$$

where q is given by

$$q(z) = \left[\frac{\beta(1-\lambda)}{\alpha}(1-z)^{\frac{2}{\alpha n}} \int_0^1 \frac{t^{\frac{\beta+\delta}{\alpha n} - 1}\alpha n}{(1-tz)^{\frac{2}{\alpha n}}}\right]^{-1} - \frac{\lambda}{1-\lambda}.$$

The function q is the best dominant.

In chapter 5, "Strong differential subordinations", the notion of strong differential subordination is defined. This notion was first introduced by J.A. Antonino and S. Romaguera in [An-Ro] and only applied by them to the special case of Briot-Bouquet differential subordinations.

Definition 5.1.1. [Ant], [An-Ro] Let $H(z, \xi)$ be analytic in $U \times \overline{U}$ and let $f(z)$ be analytic and univalent in U. The function $H(z, \xi)$ is strongly subordinate to $f(z)$, written $H(z, \xi) \prec\prec f(z)$, if for each $\xi \in \overline{U}$, the functions of zm $H(z, \xi)$ is subordinate to $f(z)$.

Remark 5.1.1. a) Since $f(z)$ is analytic and univalent, Definition 5.1.1 is equivalent to $H(0, \xi) = f(0)$ and $\mathcal{H}(U \times \overline{U}) \subset f(U)$.

b) If $H(z, \xi) \equiv H(z)$, then the strong subordination becomes the usual subordination.

In paper [GIO-GhO9] the general theory of strong differential subordinations was developed and it can be stated as follows:

Let $\Omega, \Delta \subset \mathbb{C}$, let p be analytic in U, with $p(0) = a$, $a \in \mathbb{C}$ and $\psi : \mathbb{C}^3 \times \overline{U} \to \mathbb{C}$. Implications of the following form are studied:

$$\{\psi(p(z), zp'(z), z^2p''(z); z, \xi) \mid z \in U,\ \xi \in \overline{U}\} \subset \Omega \Rightarrow p(U) \subset \Delta. \tag{5.1.1}$$

If Ω and Δ are simply connected domains with $\Omega \ne \mathbb{C}$, then (5.1.1) can be written in terms of strong subordination as

$$\psi(p(z), zp'(z), z^2p''(z); z, \xi) \prec\prec h(U) \Rightarrow p(z) \prec q(z), \tag{5.1.2}$$

where $h(U) = \Omega$, $q(U) = \Delta$, with h, q conformal mappings.

Adapted to the new notion, several definitions, lemmas and theorems used in the study of strong differential subordinations are presented.

In paragraph 5.2, the notion of linear strong differential subordinations is presented.

Definition 5.2.1. [GIO-GhO10] A strong differential subordination of the form

$$A(z, \xi)zp'(z) + B(z, \xi)p(z) \prec\prec h(z),\ \ z \in U,\ \xi \in \overline{U},\ \ A, B : U \times \overline{U} \to \mathbb{C},$$

where $A(z, \xi)zp'(z) + B(z, \xi)p(z)$ is analytic in U for all $\xi \in \overline{U}$ and $h(z)$ is analytic in U is called first order linear strong differential subordination.

Linear strong differential subordinations are studied in a disc centered in origin of radius M and in the right half-plane.

In the case of the disc, the following result was obtained:

Theorem 5.2.1. [GIO-GhO10] *Let $p \in \mathcal{H}[0, n]$, $A, B : U \times \overline{U} \to \mathbb{C}$, with $A(z, \xi)zp'(z) + B(z, \xi)p(z)$ analytic in U for all $\xi \in \overline{U}$ and*

$$\operatorname{Re}[nA(z, \xi) + B(z, \xi)] \ge 1,\ \ \operatorname{Re} A(z, \xi) > 0.$$

If

$$A(z, \xi)zp'(z) + B(z, \xi)p(z) \prec\prec Mz,\ \ z \in U,\ \xi \in \overline{U},$$

then
$$p(z) \prec Mz,\ z \in U,\ M > 0.$$

In the case of the right half-plane, the following result was obtained:

Theorem 5.2.2. [GIO-GhO10] *Let* $p \in \mathcal{H}[1,1]$, $A, B : U \times \overline{U} \to \mathbb{C}$, *with* $A(z,\xi)zp'(z) + B(z,\xi)p(z)$, *a function of* z, *analytic in* U *for all* $\xi \in \overline{U}$ *and* $\mathrm{Re}\,(z,\xi) \geq 0$, $\mathrm{Im}\, B(z,\xi) \leq n\mathrm{Re}\, A(z,\xi)$. *If*

$$\mathrm{Re}\,[A(z,\xi)zp'(z) + B(z,\xi)p(z)] > 0,\ z \in U,\ \xi \in \overline{U}$$

then $\mathrm{Re}\, p(z) > 0$, $z \in U$.

In paragraph 5.3, second order linear strong differential subordonations are presented.

Definition 5.3.1. [GIO17] A strong differential subordination of the form

$$A(z,\xi)z^2p''(z) + B(z,\xi)zp'(z) + C(z,\xi)p(z) + D(z,\xi) \prec\prec h(z),$$

where $A, B, C, D : U \times \overline{U} \to \mathbb{C}$ and $A(z,\xi)z^2p''(z) + B(z,\xi)zp'(z) + C(z,\xi)p(z) + D(z,\xi)$ is an analytic function of z for all $\xi \in \overline{U}$ and function h is analytic and univalent in U, is called second order linear strong differential subordination.

The second order linear strong differential subordinations are also studied in the case of a disc centered at origin and in the right half-plane. The case of the right half-plane was studied in paper [GIO17].

If $p \in \mathcal{H}[1,n]$ satisfies the strong differential subordination

$$A(z,\xi)z^2p''(z) + B(z,\xi)zp'(z) + C(z,\xi)p(z) + D(z,\xi) \prec\prec \frac{1+z}{1-z},\ z \in U,\ \xi \in \overline{U}$$

then $\mathrm{Re}\, p(z) > 0$, $z \in U$ and sufficient conditions for univalence are obtained for starlike functions, convex functions, close-to-convex functions and Mocanu functions by replacing:

$$p(z) = \frac{zf'(z)}{f(z)},\ p(z) = 1 + \frac{zf''(z)}{f'(z)},\ p(z) = (1-\alpha)\frac{zf'(z)}{f(z)} + \alpha\left[1 + \frac{zf''(z)}{f'(z)}\right]$$

and $p(z) = f'(z)$, $z \in U$, in the unit disc.

In paragraph 5.4, second-order non-linear strong differential subordinations are presented.

Definition 5.4.1. [GIO-GhO11] A strong differential subordination of the form

$$A(z,\xi)z^2p''(z) + B(z,\xi)zp'(z) + C(z,\xi)p(z) + D(z,\xi)p^2(z) + E(z,\xi) \prec\prec h(z),$$

where $A, B, C, D, E : U \times \overline{U} \to \mathbb{C}$, $A(z,\xi)z^2p''(z) + B(z,\xi)zp'(z) + C(z,\xi)p(z) + D(z,\xi)p^2(z) + E(z,\xi)$ is a function of z, analytic for all $\xi \in \overline{U}$ and function h is analytic and univalent in U, is called second order non-linear strong differential subordination.

Those special strong differential subordinations are studied in the case of a disc centered in origin and in the right half-plane.

The case of the right half-plane was studied in [GIO-GhO11].

If $p \in \mathcal{H}[1,n]$ and satisfies inequality

$$\mathrm{Re}\,[A(z,\xi)z^2p''(z) + B(z,\xi)zp'(z) + C(z,\xi)p(z) + D(z,\xi)p^2(z) + E(z,\xi)] > 0,\ z \in U,\ \xi \in \overline{U},$$

then
$$\mathrm{Re}\, p(z) > 0,\ z \in U.$$

From this study, sufficient conditions for univalence are obtained for starlike functions, convex functions, close-to-convex functions and Mocanu functions by replacing:

$$p(z) = \frac{zf'(z)}{f(z)},\ p(z) = 1 + \frac{zf''(z)}{f'(z)},\ p(z) = (1-\alpha)\frac{zf'(z)}{f(z)} + \alpha\left(1 + \frac{zf''(z)}{f'(z)}\right),\ p(z) = f'(z),\ z \in U,$$

in the unit disc.

Classes of functions with holomorphic functions of ξ variable as coefficient are presented in paragraph 5.5. Those classes were introduced in paper [GIO20] as follows:

Let
$$\mathcal{H}^*[a,n,\xi] = \{f \in \mathcal{H}(U \times \overline{U}) \mid f(z,\xi) = a + a_n(\zeta)z^n + a_{n+1}(\xi)z^{n+1} + \ldots,\ z \in U,\ \xi \in \overline{U}\},$$
with $a_k(\xi)$ holomorphic functions in $\overline{U}$, $k \geq n$,

$$\mathcal{H}_u(U \times \overline{U}) = \{f \in \mathcal{H}^*[a,n,\xi] \mid f(z,\xi) \text{ is univalent in } U, \text{ for all } \xi\},$$

$$A\xi_n = \{f(z,\xi) \in \mathcal{H}(U \times \overline{U}) \mid f(z,\xi) = z + a_{n+1}(\xi)z^{n+1} + \dots,\ z \in U,\ \xi \in \overline{U}\},$$

with $a_k(\xi)$ holomorphic functions in $\overline{U}$, $k \geq n+1$ and $A\xi_1 = A\xi$,

$$\mathcal{H}_u(U) = \{f(z,\xi) \in \mathcal{H}_\xi[a,n] \mid f(z,\xi) \text{ is univalent in } U, \text{ for all } \xi \in \overline{U}\},$$

$$S\xi = \{f(z,\xi) \in A\xi_n \mid f(z,\xi) \text{ is univalent in } U, \text{ for all } \xi \in \overline{U}\},$$

$$S^*\xi = \left\{ f(z,\xi) \in A\xi \mid \mathrm{Re}\,\frac{z\dfrac{\partial f}{\partial z}(z,\xi)}{f(z,\xi)} > 0,\ z \in U,\ \xi \in \overline{U} \right\},$$

denote the class of normalized starlike functions in $\mathcal{H}(U \times \overline{U})$,

$$K\xi = \left\{ f(z,\zeta) \in A\xi \mid \mathrm{Re}\,\frac{z\dfrac{\partial^2 f(z,\xi)}{\partial z^2}}{\dfrac{\partial f}{\partial z}(z,\xi)} + 1 > 0,\ z \in U,\ \xi \in \overline{U} \right\},$$

denote the class of normalized convex functions in $\mathcal{H}(U \times \overline{U})$,

$$A(p)\xi = \left\{ f(z,\xi) \in \mathcal{H}(U \times \overline{U}) \mid f(z,\xi) = z^p + \sum_{k=p+1}^{\infty} a_k(\xi)z^k,\ p \in \mathbb{N},\ z \in U,\ \xi \in \overline{U} \right\},$$

and $A(1)\xi = A\xi$.

In paper [GIO24], non-linear strong differential subordinations of the following form were studied:

$$A(z,\xi)zp'(z) + B(z,\xi)p(z) + C(z,\xi)p^2(z) + D(z,\xi) \prec\prec h(z),$$

where $A(z,\xi)zp'(z) + B(z,\xi)p(z) + C(z,\xi)p^2(z) + D(z,\xi)$ is a function of z, analytic for all $\xi \in \overline{U}$, and function h is analytic and univalent in U.

From the following non-linear strong differential subordination studied in the right half-plane

$$\mathrm{Re}\,[A(z,\xi)zp'(z) + B(z,\xi)p(z) + C(z,\xi)p^2(z) + D(z,\xi)] > 0$$

which implies $\mathrm{Re}\,p(z) > 0$, sufficient conditions for univalence for starlike functions, convex functions, Mocanu functions (α-convex), close-to-convex functions are obtained by replacing

$$p(z) = \frac{zf'(z)}{f(z)},\ p(z) = 1 + \frac{zf''(z)}{f'(z)},\ p(z) = \frac{(1-\alpha)zf'(z)}{f(z)} + \alpha\left(1 + \frac{zf''(z)}{f'(z)}\right)$$

and $p(z) = f'(z)$, $z \in U$, in the unit disc.

In paragraph 5.6, the study of Briot-Bouquet strong differential subordinations is presented. The theorems used in the development of this study are proved and dominants and the best dominant for such strong differential subordinations are given.

First, strong differential subordinations published in [GIO22] and obtained by using the following integral operators are presented:

$$L_\gamma^m f(z,\xi) = \frac{\gamma+1}{z^\gamma}\int_0^z L_\gamma^{m-1} f(t,\xi)t^{\gamma-1}dt$$

and

$$H^m f(z,\xi) = \frac{p+1}{z}\int_0^z H^{m-1} f(t,\xi)dt.$$

In chapter 6, "Tare superordonări diferenţiale", the results obtained in paper [GIO16] are presented. In this paper, I have introduced for the first time the notion of strong differential superordination as a dual concept to that of strong differential subordination following the pattern of introducing the notion of differential superordination given by professors Miller and Mocanu in [Mi-Mo10]. The general form of this theory can be presented shortly as follows:

Let Ω and Δ be any sets in $\mathbb{C}$, let p be analytic in the unit disc U, and let $\varphi : (r,s,t;\xi) : \mathbb{C}^3 \times U \times \overline{U} \to \mathbb{C}$.

In developing this theory, implications of the following form are studied:

$$\Omega \subset \{\varphi(p(z), zp'(z), z^2p''(z); z, \xi) :\ z \in U,\ \xi \in \overline{U}\} \Rightarrow \Delta \subset p(U). \tag{6.1.1}$$

If Ω and Δ are simply connected domains with $\Omega, \Delta \neq \mathbb{C}$, then there are the conformal mapings $q : U \to \Delta$, $q(U) = \Delta$, $q(0) = p(0)$ and $h : U \to \Omega$, $h(U) = \Omega$, $h(0) = \varphi(p(0), 0, 0; 0, \xi)$. If in addition, the functions p and $\varphi(p(z), zp'(z), z^2p''(z); z, \xi)$ are univalent in U, then (6.1.1) can be rewritten as

$$h(z) \prec\prec \varphi(p(z), zp'(z), z^2p''(z); z, \xi) \Rightarrow q(z) \prec p(z),\ z \in U. \tag{6.1.2}$$

Remark 6.1.1. The implication (6.1.2) also has meaning if h and q are analytic and not necessarily univalent.

Next, the definitions, lemmas and theorems used in the development of this theory are given. They were introduced in the same paper [GIO16].

I mention here the definition of the class of admissible functions.

Definition 6.1.1. [GIO16] Let Ω be a set in $\mathbb{C}$ and $q \in \mathcal{H}[a, n]$, with $q'(z) \neq 0$. The class of admissible functions denoted by $\Phi_n[\Omega, q]$, consists of those functions $\varphi : \mathbb{C}^3 \times U \times \overline{U} \to \mathbb{C}$ that satisfy the admissibility condition:

$$\varphi(r, s, t; \zeta, \xi) \in \Omega \tag{A'}$$

where $r = q(z)$, $s = \dfrac{zq'(z)}{m}$

$$\mathrm{Re}\left[\frac{t}{s} + 1\right] \leq \frac{1}{m}\mathrm{Re}\left[\frac{zq''(z)}{q'(z)} + 1\right],$$

$z \in U$, $\xi \in \overline{U}$, $\zeta \in \partial U \setminus E(q)$ and $m \geq n \geq 1$. When $n = 1$, we write $\Phi_1[\Omega, q] = \Phi[\Omega, q]$.

If $\varphi : \mathbb{C}^2 \times U \times \overline{U} \to \mathbb{C}$, then the admissibility condition (A') reduces to

$$\varphi\left(q(z), \frac{zq'(z)}{m}; \zeta, \xi\right) \in \Omega \tag{A''}$$

where $z \in U$, $\zeta \in \partial U \setminus E(q)$, $\xi \in \overline{U}$ and $m \geq n \geq 1$.

Theorem 6.1.1. [GIO16] *Let $\Omega \subset \mathbb{C}$, $q \in \mathcal{H}[a, n]$ and let $\varphi \in \Phi_n[\Omega, q]$. If $p \in Q(a)$ and $\varphi(p(z), zp'(z), z^2p''(z); z, \xi)$ is univalent in U, for all $\xi \in \overline{U}$, then*

$$\Omega \subset \{\varphi(p(z), zp'(z), z^2p''(z)); z \in U,\ \xi \in \overline{U}\}$$

implies

$$q(z) \prec\prec p(z),\ z \in U.$$

In paragraph 6.2, first order strong differential superordinations are presented.

Definition 6.2.1. [GIO15] A strong differential superordinarion of the form

$$h(z) \prec\prec A(z, \xi)zp'(z) + B(z, \xi)p(z),\ z \in U,\ \xi \in \overline{U}, \tag{6.2.1}$$

where h is analytic in U, and $A(z, \xi)zp'(z) + B(z, \xi)p(z)$ is univalent in U, for all $\xi \in \overline{U}$, is called first order strong differential superordination.

Remark 6.2.1. If $A(z, \xi) = B(z, \xi) \equiv 1$, then (6.2.1) becomes

$$h(z) \prec zp'(z) + p(z),\ z \in U$$

the differential superordination studied in [Mi-Mo10].

If $A(z, \xi) \equiv 1$ and $B(z, \xi) \equiv 0$, then (6.2.1) becomes

$$h(z) \prec zp'(z),\ z \in U,$$

the differential superordination studied in [Mi-Mo10].

Next, several theorems related to this notion are presented, such as:

Theorem 6.2.1. [GIO21] *Let $\Omega \subset \mathbb{C}$, $q \in \mathcal{H}[a, n]$, $\varphi : \mathbb{C}^2 \times U \times \overline{U} \to \mathbb{C}$ and suppose $\varphi(q(z), tzq'(z); \zeta, \xi) \in \Omega$, for $z \in U$, $\zeta \in \partial U \setminus E(q)$, $\xi \in \overline{U}$ and $0 < t < \dfrac{1}{n} \leq 1$. If $p \in Q(a)$ and $\varphi(p(z), zp'(z); z, \xi)$ is univalent in U, for all $\xi \in \overline{U}$, then*

$$\Omega \subset \{\varphi(p(z), zp'(z); z),\ z \in U,\ \xi \in \overline{U}\}$$

implies

$$q(z) \prec p(z).$$

In paragraph 6.3 different strong differential superordinations are presented which were obtained using as method for the proofs Loewner subordination chains method.

Definition 6.3.1. [GIO18] The function $L : U \times \overline{U} \times [0, \infty) \to \mathbb{C}$ is a strong subordination (or a Loewner) chain if $L(z, \xi; t)$ is analytic and univalent in U for $\xi \in \overline{U}$, $t \geq 0$, $L(z, \xi; t)$ if continuously differentiable on $\mathbb{R}^+$ for all $z \in U$, $\xi \in \overline{U}$ and $L(z, \xi; s) \prec\prec L(z, \xi; t)$, where $0 \leq s \leq t$.

Lemma 6.3.1. [GIO18] *The function $L(z, \xi; t) = a_1(\xi, t) + a_2(\xi, t)z^2 + \ldots$, with $a_1(\xi, t) \neq 0$, for all $\xi \in \overline{U}$, $t \geq 0$ and $\lim_{t\to\infty} |a_1(\xi, t)| = \infty$, is a strong subordination chain if*

$$\operatorname{Re} z \cdot \frac{\partial L(z, \xi; t)/\partial z}{\partial L(z, \xi; t)/\partial t} > 0, \ z \in U, \ \xi \in \overline{U}, \ t \geq 0.$$

In this paper, strong differential superordinations were studied using functions from the classes of functions defined in [GIO20].

Theorem 6.3.1. [GIO18] *Let $q(z, \xi) \in \mathcal{H}^*[a, 1, \xi]$, let $\varphi : \mathbb{C}^2 \times U \times \overline{U} \to \mathbb{C}$ and let*

$$\varphi(q(z, \xi), zq'(z, \xi)) \equiv h(z, \xi), \ z \in U, \ \xi \in \overline{U}.$$

If $L(z, \xi; t) = \varphi(q(z, \xi), tzq'(z, \xi))$ is a strong subordination chain, and $p \in \mathcal{H}^*[a, 1, \xi] \cap Q_\xi$, then

$$h(z, \xi) \prec\prec \varphi(p(z, \xi), zp'(z, \xi))$$

implies

$$q(z, \xi) \prec\prec p(z, \xi), \ z \in U, \ \xi \in \overline{U}.$$

Furthermore, if $\varphi(q(z, \xi), zq'(z, \xi)) = h(z, \xi)$ has an univalent solution $q(z, \xi) \in Q_\xi$, then $q(z, \xi)$ is the best subordinant.

In paragraph 6.4, using the integral operators shown in paragraph 5.7, strong differential superordinations of the following form are presented.

Theorem 6.4.1. [GIO22] *Let $h(z, \xi)$ be convex in U for all $\xi \in \overline{U}$, $h(0, \xi) = a$. Suppose that the differential equation*

$$q(z, \xi) + \frac{zq'_z(z, \xi)}{q(z, \xi)} = h(z, \xi), \ z \in U, \ \xi \in \overline{U}$$

has an univalent solution $q(z, \xi)$ that satisfies $q(0, \xi) = a$ and $q(z, \xi) \prec\prec h(z, \xi)$.

If $p(z, \xi) \in \mathcal{H}[a, 1] \cap Q_\xi$ and $p(z, \xi) + \dfrac{zp'_z(z, \xi)}{p(z, \xi)}$ is univalent in U for all $\xi \in \overline{U}$, $f(z, \xi) \in A\xi$, then

$$h(z, \xi) \prec\prec \frac{L^m_\gamma f(z, \xi)}{z} + \frac{z[L^m_\gamma f(z, \xi)]'_z}{L^m_\gamma f(z, \xi)} - 1$$

implies

$$q(z, \xi) \prec\prec \frac{L^m_\gamma f(z, \xi)}{z}, \ z \in U, \ \xi \in \overline{U}.$$

The function $q(z, \xi)$ is the best subordinant.

The new directions for research are presented in chapters 7 and 8 of this habilitation thesis.

In chapter 7, "Differential subordinations for non-analytic functions" the idea of extending the notion of subordination to the class of non-analytic functions belonging to classes C^1 and C^2 is presented. I intend to follow the pattern from the classical theory of differential subordinations for analytic functions. In this habilitation thesis, some of the first notions adapted are presented such as the definition of the subordination of non-analytic functions belonging to classes C^1 and C^2, the general form of the theory of differential subordination for non-analytic functions belonging to classes C^1 and C^2, the definition of the class of admissible functions for non-analytic functions, lemmas and theorems necessary in the development in the future of this theory. The results presented in this thesis are intended to be published in the near future in specialized journals published in Romania and abroad.

The last chapter of this thesis, chapter 8, "Differential superordinations for non-analytic functions", contains results related to the problem of the differential superordinations for non-analytic functions, the dual concept of differential subordinations for non-analytic functions belonging to classes C^1 and C^2. The definition of differential superordination for non-analytic functions belonging to classes C^1 and C^2 is given in the thesis, as well as the definition of the class of admissible functions for non-analytic functions belonging to classes C^1 and C^2, lemmas and theorems adapted from the classical theory of differential superordination, considered necessary in the future development of this theory. The results contained in this chapter are also intended to be published in the near future.

As future study I also consider the study of the starlikeness and convexity properties for differential and integral operators.

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