HABILITATION THESIS

NUMERICAL METHODS FOR INTEGRAL EQUATIONS

AND RELATED PROBLEMS

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Abstract

The present work represents a synthesis of my most significant achievements, both in my teaching and in my research career, which spans over a period of 20 years.

The main part of this work, **Professional and Scientific Achievements and Career Development Directions**, contains three chapters.

Chapter I, **Didactic Activity**, summarizes my philosophy on teaching, on the role of an educator, the courses that I have taught at four universities in Romania and abroad and the ways in which I have tried to constantly improve my teaching technique. After graduating with a Master's degree from Babeş-Bolyai University in 1991, I enrolled in a PhD program in Mathematics at the University of Iowa, in Iowa City, IA, USA. The scholarship that I received required that I teach several courses while being a PhD student. Subsequently, I held the position of Assistant Professor of Mathematics at Western Oregon University, in Monmouth, OR, USA (1997 – 2000) and at Truman State University in Kirksville, MO, USA (2000 – 2004). Since 2004, I have worked at Babeş-Bolyai University, in Cluj. During all this time, I have taught many courses at different level (Bachelor, Master, PhD), I have guided and supervised students in their learning activity, in research projects, or in writing their dissertation theses. I have also authored or coauthored a few textbooks ([39, 53, 65, 19]) and didactic papers ([50, 62, 63, 77, 52, 64]).

Chapter II, **Research Activity**, consists of four sections. Section 1, *Introduction*, gives a summary of my research interests over the years and the main areas that I studied and in which I worked and published results. In my PhD thesis ([58]), under the supervision of Prof. Kendall E. Atkinson from the University of Iowa, USA, I investigated numerical methods for the solution of the radiosity equation, obtaining convergence results, using spline collocation methods based on interpolation. Afterwards, I continued working on finding numerical solutions for several types of integral equations, through various methods. I have also applied some of these ideas to integral equation reformulations for boundary value problems.

Section 2, *Collocation methods*, contains results obtained via collocation. In [61], collocation based on interpolation of arbitrary degree r is used to obtain superconvergence results for general linear Fredholm integral equations on a smooth or piecewise smooth bounded surface $S \in \mathbb{R}^3$. Collocation based on piecewise constant interpolation (the *centroid rule*) for Fredholm-Hammerstein equations in two variables, was discussed in [49]. Also, for one-dimensional Hammerstein equations, collocation based on wavelets was recently explored in [60].

In recent years, I also considered using fixed point theory for not only proving existence and uniqueness of the solution, but also for finding numerical solutions of integral equations and giving error estimates. Rewriting an integral equation in operator form reduces the problem to finding a fixed point for the integral operator. Afterwards, some type of numerical integration has to be used, since most of

the subintegrals involved in the iterates cannot be computed exactly. These results are described in Section 3, *Iterative methods based on fixed point theory*. Picard iteration was used for Fredholm-Volterra integral equations ([47]), Volterra functional equations ([45]) and Fredholm fractional integral equations ([44]). Mann iteration (which converges faster) was employed for Volterra equations with delayed arguments ([48]) and Fredholm integral equations with modified arguments ([46]). As for the numerical integration scheme, it was found that the trapezoidal rule performed well enough. In [44], product integration was used to numerically approximate singular integrals.

Section 4, *Boundary value problems*, contains approximation results obtained using numerical methods for integral equations applied to integral reformulations of various boundary value problems. Neumann and Dirichlet problems for the Laplace equation can be solved by representing the solution as a single or a double layer potential, respectively, in which case, using a limiting argument, one obtains integral equations of the second kind. A piecewise constant collocation method is applied to a reformulation of Neumann problems on surfaces ([57]).

Nonlinear Riemann-Hilbert problems have many applications in the modeling of potential flow through pipes having porous boundaries, which are used in filtration devices, so finding a solution is of great interest. They can be reformulated as singular integral equations $A(u) = 0$, where the operator A is defined via a Nemytski operator. The operator A is then approximated by a *Fredholm quasiruled mapping* A^n. Then, employing degree of mapping theory, one can show the existence of the solution of the fully discrete equations $A^n u = 0$, for n large enough and its convergence. Linear spline approximation was used for a nonlinear Riemann-Hilbert problem on the unit disk ([67]) and trigonometric collocation was investigated to approximate the solution of a nonlinear Riemann-Hilbert problem with bounded and closed restriction curves, on the annulus ([66]).

Plans for future work are described in Chapter III, **Career Development Directions**. In Section 1, *Teaching future directions*, I mention ideas I plan on implementing in my didactic career, such as encouraging more students to actively participate in discussions and workgroups, in seminars and laboratories; reediting the textbook *Probability and Statistics for Computational Sciences*, to include more applications and more attractive examples from real life problems, especially from the area of Computer Science; writing a textbook for the course *Statistical Computational Methods*, which I have recently started teaching at several Master's programs in Computer Science; proposing a new course for the Master's in Mathematics program, called *Integral equations and applications*, which should contain the classification, the main theory and numerical methods for solving integral equations, as well as some of the many applications they have in modeling various phenomena and real life problems; this could lead to projects and topics for their dissertation theses; proposing topics for Bachelor or dissertation theses in the area of computer simulations of random phenomena; participating myself in more exchange activities with other universities, through the Erasmus program, etc.

In Section 2, *Research future directions*, I discuss areas which I plan to explore: more general nonlinear equations, like going to higher dimensions; more elaborated kernel functions, such as singular or weakly singular kernels, kernels with modified or delayed arguments; more collocation methods based on wavelets (other than Haar wavelets), two- or three-dimensional wavelets (these are of special interest, since they have many applications in Statistics, as well); develop fixed point results, collocation and iterative methods for fractional integral equations in one or two dimensions.

I also plan to investigate numerical methods for mixed Volterra-Fredholm integral equations, which arise from various formulations of the heat equation, with Neumann, Dirichlet or mixed boundary conditions, if the solution is sought in the form of a single or double layer potential. Collocation methods with interior nodes are especially convenient for such equations, if the domain has corners or sharp edges.

I plan to study numerical methods for integral equations of the third kind. Such integral equations contain a variable coefficient, multiplying the identity operator and vanishing at a number of points in the domain of definition of the equation. Integral equations of the third kind are widely investigated in theory and used in many applications, in elasticity, neutron transport, particle scattering, etc.

Also, of great interest to me are integral equations of the fourth kind (or integral algebraic equations (IAE's)). These naturally arise in many mathematical modeling processes such as problems of identification of memory kernels in heat conduction and viscoelasticity, diffusion of chemicals, evolution of a chemical reaction within a small cell, dynamic processes in chemical reactors and many more.

Recently, I started (and plan to continue) working on using cubic spline approximation in the numerical integration of partial differential equations in fluid flow problems. The dimensionless equations – obtained from equations governing the behavior of various quantities of interest (vorticity, temperature, concentration, etc.) – are discretized using the false transient technique and they are solved numerically with the spline alternating direction implicit (SADI) method.

Other future plans involve writing more applicative papers, especially in Statistics; having more collaborations with researchers from several areas (from here and abroad), who need more and stronger statistical procedures in their activity; proposing research grants on such applicative problems, that could involve young researchers, PhD students, Master's students or even students at the Bachelor level; writing a textbook on approximation methods, modeling, examples and applications.

The last part, **References**, contains 81 titles of books and papers that are referenced in the text.