
Time-Varying Linear Systems: Qualitative and Quantitative Analysis

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Habilitation Thesis Abstract

2020

Abstract

This habilitation thesis contains a synthesis of the scientific results achieved after obtaining my Ph.D in Mathematics at the West University of Timisoara, in 2012.

The thesis is organized in two main parts.

- The first part, which represents the scientific part summarizes some of the author's published results. It starts with my personal bibliography since my PhD, summing 35 articles. The results presented in the scientific part have been selected from 14 research articles. From these, many have been published in ISI journals, like Journal of Computational and Applied Mathematics, Mathematical Methods in the Applied Sciences, Applied Mathematics and Computation, Automatica J. IFAC, Journal of The Franklin Institute Engineering and Applied Mathematics.
- The second part includes a perspective plan regarding the scientific research and the professional career of the author.

Various fundamental elements of linear time-varying (LTV) systems, from both deterministic and stochastic point of view represent the heart of this thesis. The achievement of this thesis is twofold:

- the qualitative theory of linear time-varying discrete-time systems defined on the semi-axes;
- to formulate H_2 optimal filtering conditions for stochastic linear systems.

Following this, the scientific part of this thesis is organized in two chapters, which correspond to these research directions.

Chapter 1. LTV Systems: Deterministic Approach

The first chapter is devoted to the study of the so-called one-side LTV discrete-time systems. In section 1.2 it is investigated the notion of exponential stability in a more general setting, a generalized one for LTV discrete-time systems defined only on the semi-axes. A simple and practical example illustrating the relationship between the considered concept and the classical one of uniform exponential stability is presented. Also, generalized exponential stability analysis of a LTV system is carried out in terms of Lyapunov sequences. The application of such a sequence to the dual system is also established. Section 1.3 focuses on the counterpart results for the nonuniform exponential stability.

In section 1.4 are considered various concepts of exponential splitting for systems whose right-hand sides are not supposed to be invertible and the splitting concepts studied use the evolution operators in forward time. These concepts use two ideas of projections sequences: invariant and strongly invariant for the respective discrete-time system, although, in case of invertible systems, they are equivalent. These two types of projections sequences are distinct

even in the finite dimensional case. Subsection 1.4.2 deals with the so-called (h, k) –splitting concepts, subsection 1.4.3 with nonuniform concepts, respectively. It is offered a complete characterizations for these concepts and connections between them. Finally, in subsection 1.4.4 characterizations of nonuniform concept in terms of Lyapunov functions for the general case of noninvertible LTV discrete-time systems are provided. As a particular case it is considered the concept of (nonuniform) exponential dichotomy for the systems which are invertible in the unstable directions. In this way, this study is related only to the case of noninvertible systems.

Section 1.5 focuses on the trichotomic behaviour. In subsection 1.5.1, motivated by the lead given by the generalized concepts considered above, it is introduced the generalized exponential trichotomy concept. For the beginning it is shown how the mutual orthogonality property matrix projections can be replaced for the case of the generalized exponential trichotomy property. Also, these characterizations are extended for the case of invariant projection sequences. Lastly, there are presented the applications for the dual system of the developed conditions. Next, in subsection 1.5.2 are presented necessary and sufficient criteria for the class of h –trichotomy. Finally, in subsection 1.5.3 it is provided an answer to an important problem that lies within the main interest in the asymptotic behavior of the solution of LTV systems, i.e. the characterization of the trichotomy in terms of dichotomies. This approach deals with one-side LTV discrete-time systems. For these, it is considered a concept of (h, k, μ, ν) –trichotomy and it is shown how the system can be characterized in terms of two coupled systems having a (h, μ, ν) –dichotomy.

Chapter 2. LTV Stochastic Systems: Filtering Problems

Section 2.2 focuses on the problem of optimal \mathcal{H}_2 filtering for a class of continuous-time periodic stochastic systems with periodic sampled measurements. The class of admissible filters consists of deterministic continuous-time periodic systems with finite jumps. The optimal solution of the considered optimization problem is obtained by integrating a suitable generalized continuous-time Riccati equation with finite jumps. To illustrate the proposed filtering strategy, it is considered in subsection 2.2.1 a problem of field monitoring where sensors are distributed on a rectangular region in order to estimate the state of a diffusion process. It is assumed that the sensors and the filter communicate over a communication channel and the communication channel induces some communication constraints. Finally, subsection 2.2.2 treats the autonomous case.

Section 2.3 considers the problem of optimal \mathcal{H}_2 filtering for a class of continuous-time stochastic systems without assuming their exponential stability in the mean square sense. Indeed, the stability assumption is relaxed and it is assumed instead that the Lyapunov operator associated to the dynamical stochastic system is exponentially dichotomic. This section with the periodic case of the proposed problem, in subsection 2.3.2.

Chapter 3. Perspectives contains two sections.

The first one, *scientific career development plan* discusses areas which I am going to explore. There are highlighted short-term research projects like: Quasilinear LTV systems, tempered stochastic approach for LTV systems, filtering problems and linear stochastic systems with jumps. A unitary treatment of this final subject, will represent a research monograph entitled "Robust Control of Linear Stochastic Systems with Jumps. Applications to Sampled Data Control" in collaboration with V. Drăgan and S. Aberkane, accepted in the book series Lecture Notes in Control and Information Sciences Series, Springer. Further are presented three long-time research projects: computational modelling, stochastic games and industrial mathematics.

Besides theoretical research pointed out before, in the last five years I was involved in

two research programs developed between University of Alba Iulia and IPEC S.A. (a ceramic company). These are Computational Models for Reproducing Ceramics Colours (PN-III-P2-2.1-PED-2016-1835), respectively Intelligent system based on machine learning and artificial vision for the optimization of manufacturing porcelain (PN-III-P2-2.1-BG-2016-0333). From a mathematical point of view, the robust engineering method developed in this direction represents a key issue in the colour optimization process, a bridge-builder for the industrial mathematics project mentioned above.

The second part, *teaching career development plan* points out ideas that I intend to implement in my didactic career. Although the national academic development plan only provide research criteria and do not include any criteria regarding the teaching activities consider that the latter should not be minimized. As students recognition of my teaching activities, I received the Excellence Diploma Professor Bologna 2017 award granted by National Alliance of Student Organizations in Romania (ANOSR). Regarding further development, in the short term, two monographs on didactics of mathematics and arithmetics are in progress, i.e. Methods for solving arithmetic problems and Methodology of mathematical activities. These will contribute to improve my activities with my students, especially for the Pedagogy of primary and preschool education specialization. Also, Erasmus+ exchange visits will contribute to improve my lectures and to assimilate the new elements regarding the teaching activities.

The last part of this thesis, References, contains 106 titles of paper and books that are referenced in the text.